# Three Theories of Natural Rate Dynamics *

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**Abstract**

The natural interest rate is the real rate that would prevail in the long-run. The standard view in macroeconomics is that the natural rate depends exclusively on structural factors such as productivity growth and demographics. This paper challenges this view by discussing three alternative, and complementary, views: (i) that the natural rate depends on fiscal policy via the stock of risk-free assets; (ii) that it depends on monetary policy via the central bank inflation target; and (iii) that it depends on persistent supply shocks such as tariffs or wars. These three theories share the relevance of precautionary savings motives. We conclude by drawing some lessons for monetary policy design.

*Keywords*: HANK model, monetary-fiscal interactions, deep learning, cost-push shocks.

*JEL codes*: E32, E58, E63

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# 1 Introduction

The natural rate of interest, or $r^*$, is generally defined as the risk-free real interest rate that would prevail in the absence of nominal rigidities and business cycle fluctuations.[1] The natural rate concept dates back at least to Wicksell (1898). This concept is important in macroeconomics for several reasons. First, it provides a benchmark for the monetary policy stance in the long-run. Second, it plays an important role in asset pricing, as the long segment of the (real) yield curve is the sum of the natural rate and the term premium. Third, it matters for debt-sustainability analysis.[2]

The traditional view is that the natural rate primarily depends on structural factors, such as demographics and productivity growth.[3] For instance, in an economy with perpetual-youth individuals, the natural rate $r^*$ is equal to the steady-state interest rate $r^* = g^\gamma/\beta\eta - 1$,where $g$ is total factor productivity (TFP) growth, $\beta$ is the subjective discount factor, $\gamma$ denotes risk aversion , and $\eta$ is the survival probability.

In this paper we challenge this view.[4] We consider three alternative, and complementary, theories of natural-rate determination. These theories link the natural rate to fiscal policy, monetary policy and persistent supply shocks such as tariffs or wars.

First, in Section 2 we reproduce the results in Campos et al. (2024), who consider a standard heterogeneous-agent New Keynesian (HANK) model with a fiscal block and an occasionally-binding zero lower bound (ZLB). The models incorporates a continuum of atomistic households subject to idiosyncratic risk that can save only by using non-state-contingent instruments. One important feature of HANK models is the fact that the natural rate depends on the stock of public debt, as originally pointed out by Aiyagari and McGrattan (1998), and more recently by Rachel and Summers (2019) and Bayer et al.

[1]This definition roughly coincides with the natural rate definition in Obstfeld (2023), though he employs the term $\bar{r}$.

[2]There are different methods for estimating the natural rates, including semi-parametric methods (such as Laubach and Williams, 2003, or Holston et al., 2017), non-structural time series methods (Lubik and Matthes, 2015), or methods based on extracting information on the expected long-run real interest rate from bond prices (e.g., Christensen and Rudebusch, 2019, or Davis et al., 2023).

[3]See, for instance, Cesa-Bianchi et al., 2022, Gagnon et al., 2021, or Del Negro et al., 2017.

[4]A recent empirical challenge comes from Rogoff et al. (2024), who find that for most of history, long-term real interest rates have trended counter to growth and demographics.

(2023).[5] The idea is straightforward: given market incompleteness, the stock of public debt determines how much households can self-insure against negative idiosyncratic shocks and, therefore, the interest rate at which the savings market clears.[6]

The link between debt and the natural rate leads to a form of monetary-fiscal interaction. The economy has a unique steady state in which the treasury's debt target pins down the natural rate: the higher this target, the higher the natural rate is. Steady-state inflation deviates from the central bank's target proportionally to the difference between the natural rate and the intercept of the Taylor rule. Thus, to ensure price stability, the central bank should change the Taylor rule intercept depending on the treasury's long-run debt target. We develop an analytical expression linking deviations of long-term inflation from the central bank's inflation target to the policy gap between the natural rate and the central bank's long-term rate implicit in its reaction function. We evaluate this expression using market data on long-term interest rates and inflation expectations and find significant policy gaps, especially in the post-pandemic period.[7]

There is nevertheless a situation in which the central bank in a HANK world cannot deliver on its long-run mandate even if it is willing to adapt its rule to the fiscal position. If the debt level is sufficiently low, the natural interest rate becomes negative. If the natural rate is so negative that its sum with the inflation target falls below zero, the ZLB will be binding in the long term and inflation will be equal to the opposite of the natural rate. Thus, there is a minimum debt level that is compatible with the inflation target. For any debt target below that minimum level, the central bank fails to deliver on its

[5] Bayer et al. (2023)and Hansel (2024) analyze how a permanent increase in the ratio of public debt to GDP increases real public bond yields in the long run. Hagedorn (2016) shows that prices and inflation are jointly and uniquely determined by fiscal and monetary policy in a HANK model with nominal debt. Kaplan et al. (2023) analyze the fiscal theory of the price level (FTPL) in the context of a heterogeneous-agent model with flexible prices. In their model, a permanently higher deficit is associated with a lower steady-state real interest rate and less real public debt, as well as a higher long-run inflation rate for a given monetary policy setting.

[6] The natural rate is the real interest rate in the deterministic steady state of the model.

[7] Chortareas et al. (2023) estimate a time-varying Taylor rule for the United States and document how the Federal Reserve has occasionally misread the natural rate of interest. Bocola et al. (2024) use high-frequency data to detect shifts in financial markets' perception of the Federal Reserve stance on inflation.

mandate.[8]

Second, we follow Fernandez-Villaverde et al. (2023) and build a HANK model with a ZLB and aggregate demand shocks in Section 3. The model is similar to the one above, but we solve it using global methods to capture how households' demand for savings increases with the frequency of ZLB spells. This is due to a precautionary motive: when the nominal rate is constrained by the ZLB the central bank cannot provide an adequate degree of monetary accommodation: households then save to self-insure against aggregate risk linked to the ZLB.[9] The increased demand for precautionary savings lowers the natural interest rate. Since the long-run nominal interest rate is simply the real interest rate plus inflation, the average nominal rate is lower, providing less room for the central bank to stabilize the economy away from the ZLB. This creates a feedback loop through which lower natural rates increase the frequency of ZLB episodes, making it more likely to reach the ZLB, thus further reducing the natural rate.

This observation has a significant implication: monetary policy is not neutral in the long run in a HANK economy, as the Fisher equation depends on the central bank's stance. More specifically, if we denote the steady-state nominal interest rate, inflation rate, and inflation target by $i$, $\pi$, and $\bar{\pi}$, respectively, then $i(\bar{\pi}) = r^*(\bar{\pi}) + \pi(\bar{\pi})$, such that $dr^*/d\bar{\pi} > 0$, meaning that the natural rate increases with the central bank inflation target.

Third, we analyze the representative-agent New Keynesian (RANK) model with persistent supply shocks of Nuno et al. (2024) in Section 4. The novelty of that model lies in the introduction of a persistent cost-push shock in addition to the standard autoregressive ‘”shocks. The economy randomly switches between two regimes: ‘times”, when the persistent shock does not affect the economy, and ‘times”, when the shock increases production costs for all firms. This can be interpreted as a proxy for various types of cost-increasing

[8]Mian et al. (2022) show that greater deficits may reduce, rather than increase, debt when at, or near, the ZLB.

[9]This mechanism is already present in representative-agent models (e.g., Adam and Billi, 2007, Nakov, 2008, and Bianchi et al., 2021). Fernandez-Villaverde et al. (2023) and Schaab (2020) show that heterogeneity makes it more acute.

shocks. For example, Afrouzi et al. (2023) consider a shock similar to the one used here as a proxy for changes in labor market composition towards more regulated labor sources or a deceleration in globalization.[10]

There is a precautionary-savings mechanism through which the persistent cost-push shock affects the natural rate. During normal times, the economy is undistorted, and consumption aligns with that of efficient allocation; thus, the output gap is zero. In contrast, during bad times the average markup becomes suboptimal, leading to a reduction in output and consumption which results in a negative output gap. Consequently, the economy features two distinct stochastic steady states (SSSs), each associated with a different regime.[11] The variation in consumption between the two regimes explains the dynamics of the natural interest rate.[12] In normal times, households anticipate a potential transition to a regime where their average consumption would decline, leading to a precautionary increase in the demand for savings. Given the fixed supply of savings instruments, this results in a decline in the natural rate. Conversely, when the economy transitions to the bad times regime, consumption falls, and the demand for savings decreases as households anticipate higher future consumption once the regime ends, causing the natural rate to rise.

Traditional Taylor rules fail to stabilize inflation in both regimes due to shifts in the natural interest rate. Specifically, the Taylor rule sets the long-term real interest rate equal to the average natural rate across regimes, which proves too restrictive during normal times and too lax during bad times. Consequently, inflation systematically deviates from the target, underscoring the limitations of existing policy rules that assume a constant natural rate.

What these three theories have in common is the central role played by precautionary savings motives. In the three models, either agents react to shifts in idiosyncratic or

[10] This addition complicates the model solution, as it requires solving the model globally with regime-switching dynamics.

[11] An SSS refers to the equilibrium state when shocks are zero, and the economy remains within its current regime, with agents anticipating the stochastic processes.

[12] The natural interest rate is defined here as the real interest rate in each SSS corresponding to the long-term real interest rate when temporary shocks dissipate.

aggregate risks by changing their demand for risk-free saving instruments, or the government changes the long-run supply of these instruments. As the natural rate is the inverse of the price of risk-free assets, these changes lead to different levels of the natural rate. Compared to the standard complete-markets representative-agent economy in which the demand for assets is perfectly elastic, the presence of precautionary savings gives rise to an upward-sloping demand.

These theories bear relevant implications for monetary policy design: (i) central banks should track, as best as possible, the future path of the natural rate, and avoid assuming that this is a slow-moving variable;[13] (ii) the natural rate may jump in response to fiscal and political events; and (iii) when deciding inflation targets, central banks should take into account that the natural rate may decrease if long-term inflation is too low, and therefore aim for a reasonable inflation target that guarantees that nominal rates remain well above the ZLB.

# 2 A fiscal theory of the natural rate

We start with the model in Campos et al. (2024), in which the natural rate depends on fiscal policy. First we introduce their model and then we describe the implications for the natural rate.

## 2.1 Model

The model is a baseline discrete-time HANK model with monetary and fiscal policy. Wages are subject to nominal rigidities and hours are determined by a union on behalf of the workers. Firms produce the final good with the labor supplied by the union. The

[13] Bauer and Rudebusch (2020) show how accounting for time variation in natural rates is crucial for understanding the dynamics of the yield curve, a key object in monetary policy transmission. Alternatively, central banks may look for monetary policy rules that are robust to natural rate dynamics. Campos et al. (2024), for instance, show how a monetary policy rule à la Orphanides and Williams (2002) may stabilize inflation and output gap when the natural rate evolves with long-term debt without any reference to the natural rate. Daudignon and Tristani (2023) analyze the optimal monetary policy response to stochastic changes in the natural rate in a New Keynesian model.

model is closed by a monetary policy authority, which determines the nominal interest rate, and a treasury, responsible for taxation, spending and public debt issuance.

**Households.** There is a continuum of households indexed by $i \in [0,1]$. Households derive utility from consumption, $c_{i,t}$, and disutility from working $n_{i,t}$ hours. They can only save in a nominal public bond. Given a discount factor $\beta$, the intertemporal problem solved by each household is:

$$
\begin{aligned}
V(a_{i,t}, z_{i,t}) = \max_{c_{i,t}, a_{i,t+1}} & \ u(c_{i,t}) - v(n_{i,t}) + \beta \mathbb{E}_t [V(a_{i,t+1}, z_{i,t+1})] \\
\text{s.t.} \quad c_{i,t} + a_{i,t+1} &= (1+r_t) a_{i,t} + (1-\tau) \frac{W_t}{P_t} z_{i,t} n_{i,t} + T_t, \\
a_{i,t+1} &\geq \bar{a},
\end{aligned}
$$

where $a_{i,t}$ is the household's asset position in real terms at the start of the period, $z_{i,t}$ is the idiosyncratic labor productivity, $r_t$ denotes the *ex-post* real return of bonds in period $t$, $W_t$ is the nominal wage, and $P_t$ is the price level. Labor income is taxed at a constant rate $\tau$. Households receive real net lump-sum transfers $T_t$ from the treasury. Households cannot short bonds, i.e. $a_{i,t+1} \geq \bar{a} = 0$.

At time $t$, household $i$ works $n_{i,t}$ hours. A union chooses these hours on behalf of households. Each hour provides $z_{i,t}$ units of effective labor, so that aggregate hours are $N_t = \int_0^1 z_{i,t} n_{i,t} di$. The idiosyncratic shock $z_{i,t}$ follows a first-order Markov chain with mean $\mathbb{E}_t z_{i,t+1} = 1$. Agents take their hours $n_{i,t}$ as given. We assume a proportional allocation rule for labor hours, with $n_{i,t} = N_t$. The nominal wage $W_t$ is determined by union bargaining as specified below.

**Unions.** The union aggregates different labor tasks provided by the households into a homogeneous labor service. The union employs all households for the same number of hours $N_t$ and sets nominal wages by maximizing the welfare of the average household subject to a penalty term on the deviation of nominal wages from the central bank's inflation target $\overline{\pi}$.

Solving this problem leads to a wage Phillips curve (see Auclert et al., 2018):

$$\log\left(\frac{1+\pi_t^w}{1+\overline{\pi}}\right) = \kappa_w\left[-\frac{\epsilon_w-1}{\epsilon_w}(1-\tau)\frac{W_t}{P_t}\int u'(c_{it})z_{it}di + v'(N_t)\right]N_t + \beta\log\left(\frac{1+\pi_{t+1}^w}{1+\overline{\pi}}\right), \tag{1}$$

where $\epsilon_w$ is the elasticity of substitution between different labor tasks, $\kappa_w$ is the slope of the Phillips curve (itself a nonlinear function of other parameters of the model), and $\pi_t^w \equiv \frac{W_t}{W_{t-1}} - 1$ is the nominal wage inflation rate.

**Firms.** There is a continuum of identical firms. Firms produce final goods using a constant return-to-scale technology $Y_t = \Theta N_t$, where $N_t$ is aggregate labor and $\Theta > 0$ is a constant productivity parameter. The real wage is given by $\frac{W_t}{P_t} = \Theta$. From these equations, wage inflation is equal to goods inflation, $\pi_t^w = \pi_t$, where the latter is defined as $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$.

**Monetary policy.** The central bank sets the nominal interest rate on nominal bonds $i_t$ according to a standard monetary policy rule that responds to inflation, and it is subject to a ZLB:

$$\log(1+i_t) = \max\left\{\log(1+\overline{r}) + \log(1+\overline{\pi}) + \phi_\pi\log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right), 0\right\}, \tag{2}$$

where $\phi_\pi \geq 1$ is the slope of the Taylor rule, $\overline{\pi}$ is the inflation target, and $\overline{r}$ is the real rate intercept.

**Fiscal policy.** The treasury collects labor taxes from households and spends on real government consumption $G_t$ and real lump-sum transfers to households $T_t$ (a negative real transfer can be interpreted as a lump-sum tax). Also, the treasury can issue one-period nominal bonds. Government consumption does not enter into households' utility (or, equivalently, it enters in a separable way from private consumption and labor supply, and we can drop the term).

Given a level of real tax collection $\mathcal{T}_t$, public debt accumulates according to:

$$P_t B_t = (1 + i_{t-1}) P_{t-1} B_{t-1} + P_t (G_t + T_t - \mathcal{T}_t),$$

where $B_t$ denotes the stock of bonds in real terms. If we define *ex-post* real rates as $(1 + r_t) \equiv (1 + i_{t-1}) \frac{P_{t-1}}{P_t}$, we can express the government's budget constraint as

$$B_t = (1 + r_t) B_{t-1} + T_t + G_t - \mathcal{T}_t.$$

Tax collection is given by:

$$\mathcal{T}_t = \int_0^1 \tau \frac{W_t}{P_t} z_{i,t} n_{i,t} di.$$

$\mathcal{T}_t$ follows an endogenous process determined by the evolution of its underlying component variables. Similarly to $\tau$, transfers $T_t = T$ will be a constant in the baseline calibration.

In comparison, government consumption, $G_t$, follows a fiscal rule depending on expenditure $\overline{G}$ and the debt target $\overline{B}$:

$$G_t = \overline{G} - \phi_G (B_{t-1} - \overline{B}), \tag{3}$$

where $0 < \phi_G < 1$ controls the speed of fiscal adjustment when debt is not at its target.

**Aggregation and market clearing.** In equilibrium, the labor, bond, and good markets clear:

$$N_t = \int_0^1 z_{i,t} n_{i,t} di,$$
$$B_t = \int_0^1 a_{i,t+1} di,$$
$$C_t = \int_0^1 c_{i,t} di,$$

and the aggregate resource constraint holds: $G_t + C_t = Y_t$.

**Calibration and computational method.** The calibration is described in Appendix 5. The model is solved only in the steady state.

## 2.2 The natural rate and the long-run debt level

We focus on the deterministic steady state (DSS) of the model. In the DSS there are no aggregate shocks, but there are idiosyncratic shocks at the household level. First we characterize the demand and the supply of bonds and then we combine supply and demand with the monetary policy rule to obtain the real interest rate and the inflation rate. We denote all steady-state variables with the subindex "ss," except for the real interest rate, for which we use the standard $r^*$, since, in the model, this variable coincides with what is usually called the long-run natural rate. In addition, we retain the $t$ subindex for variables that relate to choices made by households because they still face idiosyncratic shocks in the DSS.

**The natural rate as a function of the debt level**. The demand for bonds aggregates the individual savings decisions of households, which accumulate bonds to smooth their consumption across time and the world's idiosyncratic states. We express the aggregate demand for bonds in the DSS by

$$A_{ss}(r^*) = \int_0^1 a_{i,t+1}(r^*)\, di,$$

where we make explicit that both the aggregate and the individual demands for bonds are a function of $r^*$. The solid red line in Figure 1 displays the demand for bonds. It is well-known, at least since Aiyagari (1994), that an economy with incomplete markets such as the one analyzed here produces an upward-looking demand curve. This is due to precautionary savings: as the volume of debt decreases, households are willing to accept debt with lower returns in order to save against potential bad realizations of idiosyncratic income risk.

The supply of bonds in the DSS is exogenous and equals $\overline{B}$. The vertical black dashed line in Figure 1 displays the supply of bonds.

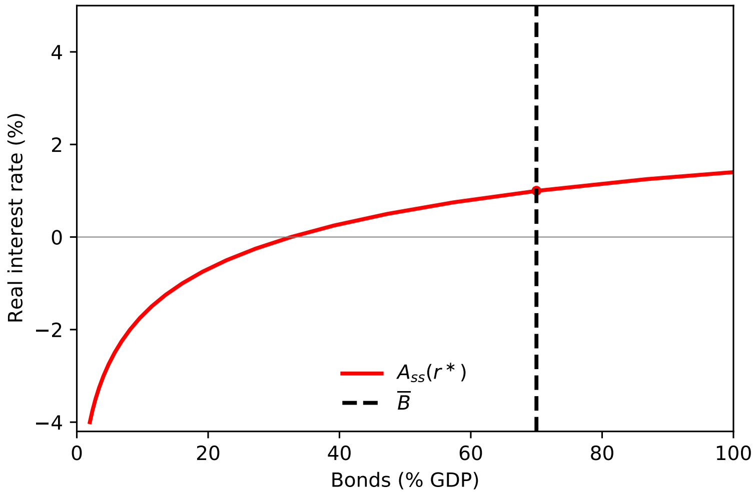


Figure 1: Determination of the natural interest rate

We combine the supply and demand of bonds and the monetary policy rule to characterize the DSS of the model. When we equate supply and demand, we find that $A_{ss}(r^*) = \overline{B}$. The vertical supply of bonds allows us to pin down the range of steady-state real interest rates that align with the steady-state debt target. Because the demand for bonds slopes upward, an increase in the steady-state debt level will result in a higher steady-state real interest rate. By inverting the function $A_{ss}(r^*)$, we can establish that the relationship $r^*\left(\overline{B}\right)$ is an increasing function. In other words, the natural rate depends positively on the level of debt.

The bottom line is that, as we deviate from complete markets, the precautionary savings motive induces a demand for bonds that is not perfectly elastic, thus making the long-term real rate (also known as the natural rate) a function of the level of long-term debt.

## 2.3 Implications for monetary policy design

**Long-term inflation rates**. We approximate the monetary policy rule (2) in the DSS by $i_{ss} \approx \max\{\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}), 0\}$. This rule can be combined with the long-run Fisher equation $i_{ss} = r^* + \pi_{ss}$ to get:

$$r^* + \pi_{ss} \approx \max\{\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}), 0\}. \tag{4}$$

It is well known that equation (4) has two solutions (Benhabib et al., 2002). The first solution, which we term a *non-binding ZLB* scenario, corresponds to the case in which $\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}) > 0$, and results in the solution $r^* + \pi_{ss} \approx \overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi})$, or equivalently

$$\pi_{ss} \approx \overline{\pi} + \frac{r^* - \overline{r}}{\phi_\pi - 1}. \tag{5}$$

which relates the deviation of long-term inflation from the inflation target $\pi_{ss} - \overline{\pi}$ to the *policy gap* $r^* - \overline{r}$ between the natural rate and the intercept in the central bank's reaction function. The second solution to equation (4) is a *binding ZLB* scenario, in which $\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}) \leq 0$, meaning that the maximum of the right-hand side of the equation is zero. In this case, the nominal interest rate is zero, and $\pi_{ss} = -r^*$.

**What happens if the central bank fails to track the natural rate?** Equation (5) implies that, if the central bank wishes to ensure that long-run inflation remains at its target, $\pi_{ss} = \overline{\pi}$, the intercept $\overline{r}$should perfectly track the natural rate $r^*$, ensuring that the policy gap is always zero. Otherwise, the central bank fails to deliver on its mandate.

To investigate whether this second possibility has some empirical support, Campos et al. (2024) take equation (5) to the data. To this end, they collect variables that approximate the (potentially time-varying) steady-state values of real interest rates and inflation. They think of steady-state objects as random variables instead of constant parameters.[14]

[14] They collect daily data on the 5-year 5-year (5y5y) forward nominal yield. This is a measure of the 5-year yield expected five years ahead, which is commonly used as a proxy for long-term nominal interest rates. For long-term inflation, they employ the 5y5y inflation-linked swaps (ILS). These are swap

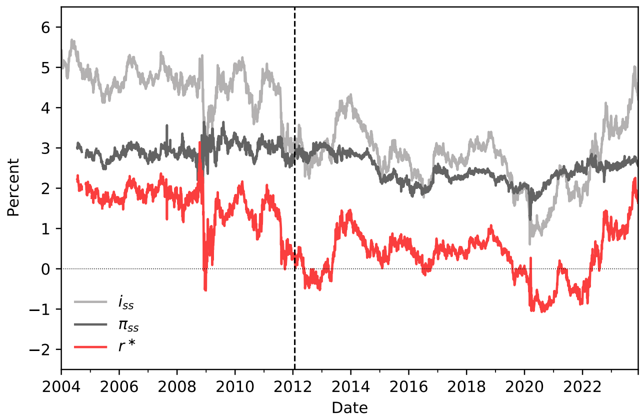


Figure 2: Long-term nominal and real rates and inflation
*Note*:$i_{ss}$ is the 5y5y forward nominal rate obtained from the zero-coupon US yield curve. $\pi_{ss}$ is the 5y5y ILS. $r^*$ is computed as the difference $i_{ss} - \pi_{ss}$. The dashed vertical line marks the date when the 2% inflation target was announced (January 24, 2012).

Their results are reported in Figure 2. Three patterns are immediately apparent. First, market expectations of long-term nominal and real rates and inflation are neither constant nor evolve solely based on low-frequency secular trends, but display a significant level of high- and medium-frequency volatility. Second, both nominal and real rates display greater volatility than inflation. Three, long-term inflation expectations have systematically deviated from the 2 % target. The Federal Reserve officially adopted this value in January 2012, but it was considered the implicit target long before that date, in line with other major central banks such as the ECB and the Bank of England.

Figure 3 plots the estimated policy gap $r^* - \bar{r}$ using equation (5).[15] This gap was markedly different from zero before 2014. From 2015 to 2020, the gap largely closed, but it reopened again after the large fiscal expansion that followed the COVID-19 pandemic. These results provide evidence supporting the idea that market participants perceive that the Federal Reserve's reaction function has not always tracked the natural rate perfectly, which explains the dynamics in long-term inflation.

**The minimum debt level compatible with an inflation target**. Next, we com-

contracts that transfer inflation risk from one party to another through an exchange of fixed cash flows. The real interest rate is computed as the difference between the 5y5y nominal rate and the 5y5y ILS.

[15]See Campos et al. (2024) for a discussion on how the slope of the Taylor rule can be estimated from market data.

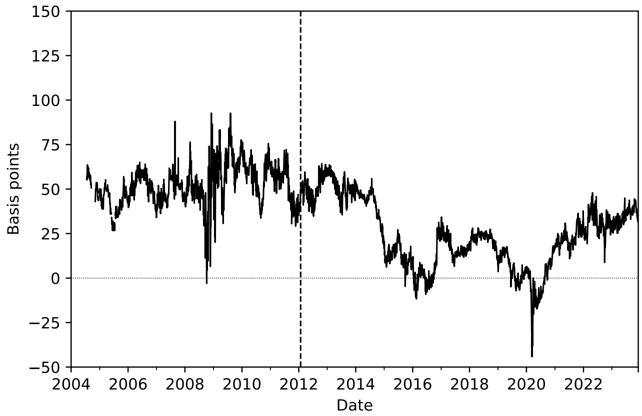


Figure 3: Policy gap $r^* - \overline{r}$

*Note*: The policy gap is based on Figure 2 .

bine the real interest rate that clears the bond market in the DSS with the Taylor rule. When the ZLB is not binding, we assume that the central bank picks this real interest rate as the intercept in the Taylor rule to ensure that inflation remains on target. That is, $\overline{r} = r^*\left(\overline{B}\right)$ and, hence, $\pi_{ss}\left(\overline{B}\right) = \overline{\pi}$. This can always be achieved if real interest rates are high enough. There is, however, a level of the debt target $\overline{B}^*$, defined as

$$r^*\left(\overline{B}^*\right) + \overline{\pi} = 0,$$

for which the nominal interest rate is zero. We illustrate this result in Figure 4. It displays the nominal interest rate for two different inflation targets, 2% (solid red) and 0% (dashed black). The Figure shows that, for an inflation target of 2%, the minimum debt level $\overline{B}^*$ is 8% of GDP. In contrast, for 0%, it is 33%.

For debt levels below $\overline{B}^*$, $\overline{B} < \overline{B}^*$, the central bank is forced to accept steady-state inflation levels above its target due to the inability of nominal rates to become negative. In this case, the ZLB is binding, $i_{ss} = 0$ and inflation is determined by $\pi_{ss} = -r^*\left(\overline{B}\right)$. Campos et al. (2024) call $\overline{B}^*\left(\overline{\pi}\right)$ the *minimum debt level compatible with the inflation target* $\overline{\pi}$, because the central bank can deliver on its inflation target in the DSS only if $\overline{B} > \overline{B}^*$. The dashed line in Figure 5 displays a frontier $\pi\left(\overline{B}^*\right)$ for positive inflation targets. The shaded area above this frontier contains the set of (non-negative) inflation

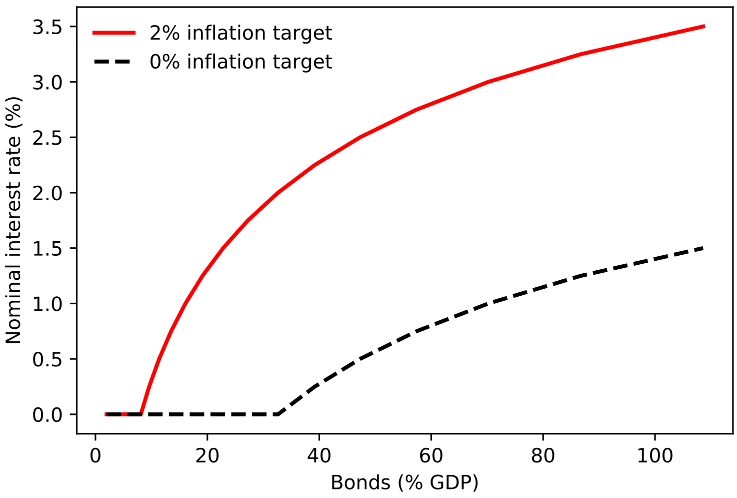


Figure 4: Steady-state nominal interest rate as a function of long-term debt
*Note*: The black dashed line shows the steady-state nominal interest rate as a function of the long-run debt level $\overline{B}$ for an inflation target of 0% and the red solid line for 2%.

targets that can be achieved in equilibrium for varying levels of debt. The level of debt $\overline{B}^*$ shown in this graph is the lowest level of debt compatible with an inflation objective of zero.

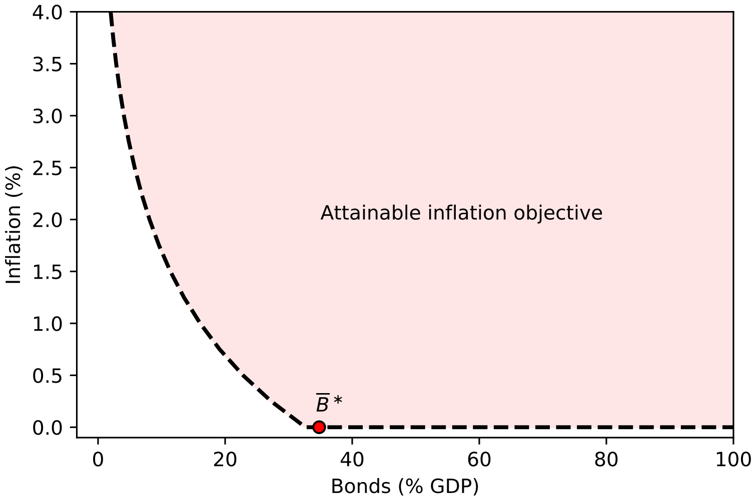


Figure 5: Steady-state inflation as a function of long-term debt
*Note*: The shaded area shows the combinations of (non-negative) inflation objectives that can be achieved in equilibrium for a varying level of debt. The level of debt $B^*$ is the lowest level of debt compatible with an inflation objective of zero.

An alternative interpretation of the minimum debt level compatible with the inflation target is that, if public debt is too low, conventional monetary policy will not be enough to guarantee the central bank inflation target, and other interventions are needed.

# 3 Inflation, the ZLB, and the natural rate

Next, we challenge the view that the natural rate is independent from monetary policy. We summarize the main results in Fernandez-Villaverde et al. (2023).

## 3.1 Model

The model is a HANK model similar to the one in Section 2. Here we highlight the main differences.

**Households.** Households maximize a similar problem as in Section 2. There are only two differences. First, households are affected by a preference shifter $\xi_t$, such that instantaneous utility is $\xi_t \left[u\left(c_{i,t}\right) - v\left(n_{i,t}\right)\right]$. The shock evolves as an AR(1) process in logs, $\log \xi_t = \rho_\xi \log \xi_{t-1} + \zeta_t$, where $\rho_\xi \in (0,1)$ and $\zeta_t \sim \mathcal{N}(0, \omega_\xi)$. Second, there are no labor taxes ($\tau = 0$): all taxes are lump sum.

**Labor market.** The wage Phillips curve is now (Hagedorn et al., 2019)

$$\log\left(\frac{1+\pi_t^w}{1+\overline{\pi}}\right) = \kappa_w \left[\frac{v'\left(N_t\right)}{u'\left(C_t\right)} - \frac{\epsilon_w - 1}{\epsilon_w}\frac{W_t}{P_t}\right] + \beta \mathbb{E}_t \left[\log\left(\frac{1+\pi_{t+1}^w}{1+\overline{\pi}}\right)\frac{N_{t+1}}{N_t}\right], \tag{6}$$

**Central bank.** The central bank sets the nominal interest rate $i_t$ according to the Taylor rule:

$$1 + i_t = \max\left\{1, \left(1+\overline{r}\right)\left(1+\overline{\pi}\right)\left(\frac{1+\pi_t}{1+\overline{\pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\widetilde{Y}}\right)^{\phi_y}\right\},$$

where $\widetilde{Y}$ denotes the level of output, both in the DSS of the economy. The central bank sets $i_t$ by reacting to inflation rate deviations from its target —the parameter $\phi_\pi$ determines the strength with which this happens—and in output deviation from its DSS level—the parameter $\phi_y$ pins down the strength of this second channel, unless the ZLB constraint $i_t \geq 0$ is binding, in which case it sets $i_t = 0$.

**Fiscal authority.** The fiscal authority levies lump-sum taxes $T_t$ on households to finance a fixed amount of outstanding debt $\widetilde{B}$, such that the government budget constraint is satisfied $T_t = r_t\widetilde{B}$. Given $\widetilde{B}$ and the equilibrium interest rate, the lump-sum taxes $T_t$ are set to clear the budget constraint.

**Calibration and computational method.** We calibrate the model to replicate labor earnings and wealth dispersion, average marginal propensity to consume and the frequency of ZLB occurrences observed in the US economy since 1945. The calibration is described in Appendix 5. We solve the model non-linearly using the neural network algorithm proposed by Fernández-Villaverde et al. (2023).[16]

## 3.2 The natural rate and the ZLB

**Ergodic distributions.** Before analyzing the link between monetary policy and the natural rate, it is important to understand the implications of this non-linearity for the model's dynamics. An intuitive way to grasp this is by comparing the ergodic distribution of the aggregate variables in the model, which we call ZLB-HANK, with those of a standard HANK economy with the same parameterization, except that the ZLB is absent from the Taylor rule followed by the central bank.

Figure 6 shows the ergodic distribution of inflation, the nominal interest rate, the real interest rate and aggregate consumption in the ZLB-HANK and HANK economies. The figure shows how the presence of the ZLB skews the dynamics of the model to the left, except for the real rate, where the left tail gets truncated. These are the cases in which the ZLB constrains the nominal interest rate, and the economy experiences a sharp drop

[16] This allows us to compute the stochastic equilibrium dynamics of the economy instead of using standard computation methods for HANK models based on linearization (see, for example, Ahn et al., 2018, and Auclert et al., 2021).

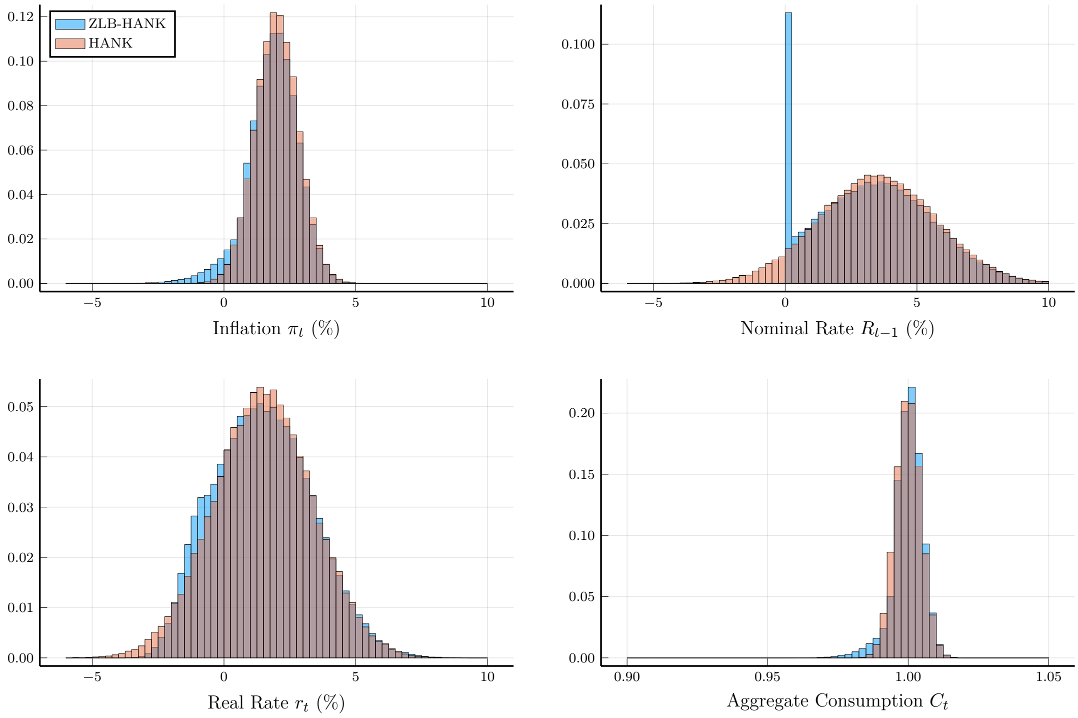


Figure 6: Ergodic Distributions

*Note*: The figure displays the ergodic distribution in the model. The figure is produced by simulating the model for a large number of periods.

in aggregate consumption amidst a deflationary spiral and real rates that are too high. These dynamics are absent in the HANK economy.

**The natural rate and the ZLB.** Standard models typically predict that structural parameters determine the real interest rate in the DSS, meaning that monetary policy does not influence it. Although this model yields similar results in the DSS, it generates a relationship between changes in the inflation target and changes in real interest rates in the stochastic steady state (SSS).[17]

For instance, in the DSS of the model, the inflation rate is 2%, which coincides with the central bank's inflation target. However, in the SSS the level of inflation is lower (1.90%). This happens because households internalize the possible occurrence of sizable negative demand shocks that could push the economy to the ZLB constraint, triggering

[17] The SSS is a point at which all aggregate variables are constant, and the realization of the aggregate demand shock is zero. However, agents are aware that non-zero realizations can come in the future and continue to face idiosyncratic risk. The SSS is an important concept because it often provides a better summary of the ergodic distribution of non-linear models than the DSS.

deflationary episodes. In the following paragraphs, we will show how this model implies that changes in the level of the central bank's inflation target move real interest rates in the SSS even if they do not do so in the DSS.

| | ZLB-HANK | | HANK | |
|---|---|---|---|---|
| Variable | DSS | SSS | DSS | SSS |
| Inflation | 2.00% | 1.92% | 2.00% | 1.99% |
| Nominal Rate | 3.50% | 3.32% | 3.50% | 3.48% |
| Real Rate | 1.50% | 1.40% | 1.50% | 1.48% |
| (Shadow) ZLB Frequency | - | 9.62% | - | (5.70%) |

Table 1: Comparison of DSS and SSS in ZLB-HANK, and HANK.

Table 1 reports the DSS and SSS values for inflation, the nominal interest rate and the real interest rate, along with the frequency of ZLB events in each model's ergodic distribution.[18]

Both the ZLB-HANK and the HANK economies exhibit identical values for all macroeconomic variables at the DSS since the ZLB is not binding at that point. However, the SSS values of the two economies diverge. Specifically, the HANK economy displays SSS values for inflation, the nominal rate, and the real rate that are virtually identical to those at the DSS, differing by only 1 bp, 2 bp and 2 bp, respectively. In contrast, introducing the ZLB into the ZLB-HANK economy reduces the SSS values for inflation, the nominal rate, and the real rate by approximately 8 bps, 18 bps, and 10 bps, respectively.

How does the introduction of the ZLB explain these differences? The ZLB changes the behavior of households and firms in the SSS, even if it is not binding at that point. Agents understand that a demand shock could push the economy toward the ZLB in the future and respond preemptively. After all, the ZLB-HANK economy spends 9.62% of quarters at the ZLB. In particular, households increase their precautionary savings to ensure a savings buffer (or reduce their borrowing) to smooth their consumption stream in recessions where the nominal rate hits zero. This effect is particularly marked for

[18]For the HANK economy, we report the shadow frequency and duration of ZLB events, defined as any period where the nominal interest rate equals or falls below zero.

wealth-poor households, which suffer disproportionally more from ZLB episodes. Higher precautionary savings exert downward pressure on the real interest rate level, reducing the central bank's room for maneuvering the nominal rates and making the ex-post realization of ZLB events even more likely.

In comparison, the shadow frequency of ZLB events in the HANK economy is lower (5.70% of quarters) since the central bank can accommodate negative demand shocks with aggressive reductions of the nominal interest rate more effectively, and households have a smaller need for precautionary behavior.

## 3.3 Implications for monetary policy design

We just saw how, in the ZLB-HANK economy, the presence of ZLB episodes changes the real interest rate even when we are not at the ZLB. This observation raises an intriguing possibility. Since the central bank can alter the frequency of the ZLB episodes by modifying its inflation target, it can affect the level of real interest rates through the change in the households' savings behavior. In other words, monetary policy is not neutral, even in the long run. More specifically, the model features a long-run Fisher equation, $i_{ss}(\bar{\pi}) = r^*(\bar{\pi}) + \pi_{ss}(\bar{\pi})$, in which the real rate in the SSS depends on the central bank's inflation target $\bar{\pi}$. In this setting, a higher inflation target raises the SSS level of the real rate, that is, $dr^*/d\bar{\pi} > 0$.

To establish this result, we compare the real interest rate levels in different model economies, which differ only in their inflation target $\bar{\pi}$. Figure 7 plots the DSS and SSS levels of inflation, the real rate, the nominal rate, and the frequency of ZLB episodes for $\bar{\pi}$ between 1.7% and 4% in both the ZLB-RANK and ZLB-HANK economies. The ZLB-RANK is a version of the model where we shut down idiosyncratic labor risk to obtain a representative household. First, the graph shows that, in both economies, the central bank successfully achieves its inflation target at the SSS when $\bar{\pi}$ falls within the range of 3% to 4%. For those high inflation targets, the probability of experiencing a ZLB event is so low that the non-linearity of the model is not quantitatively relevant. The economy

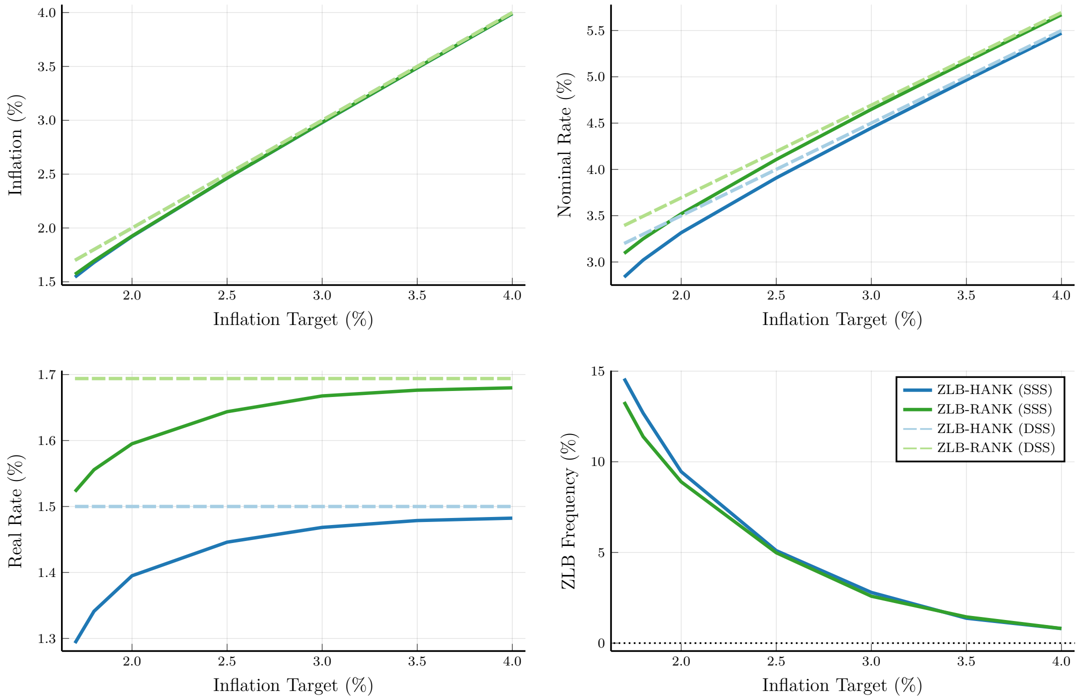


Figure 7: DSS/SSS in ZLB-RANK and ZLB-HANK as a function of the inflation target

behaves in practice as if there were no ZLB constraint, and changes in the inflation target between 3% and 4% do not alter the level of real interest rates.

However, when the inflation target falls below 3%, the non-linearity caused by the ZLB kicks in, and the SSS and DSS diverge substantially. First, the central bank undershoots its inflation target. When the target is 1.7%, the inflation rate in the SSS is 1.54%.[19] This 16 bp undershoot is associated with the fact that the economy spends 15% of quarters at the ZLB, compared with less than 1% of quarters when the inflation target is 4%. Importantly, in the ZLB-HANK economy the sensitivity of the probability of reaching the ZLB to changes in the inflation target is higher than in the ZLB-RANK economy. In the latter, the proportion of quarters spent at the ZLB goes from less than 1% when the inflation target is 4% to 13% when the inflation target is 1.7%.

As we vary the inflation target, the real interest rate in the DSS does not change (dashed lines in the bottom left-hand panel of Figure 7), since idiosyncratic labor income

[19]This undershoot is happening in the SSS, where the economy is not at the ZLB. In other words, the undershoot is caused by the agents reacting to the possibility of being at the ZLB in the future, not because of the deflationary spiral triggered when the ZLB is reached.

risk is either independent of the inflation target in the ZLB-HANK economy or non-existent in the ZLB-RANK economy. In comparison, the real rate in the SSS increases with the inflation target because we reduce the probability of hitting the ZLB (solid lines in the bottom left panel of Figure7). Notably, the sensitivity of the real rate in the SSS to changes in the inflation target is greater in the ZLB-HANK economy than in the ZLB-RANK economy (i.e. $dr^*/d\bar{\pi}$ in ZLB-HANK is greater than $dr^*/d\bar{\pi}$ in ZLB-RANK). In other words, looking at the DSS could lead to the incorrect conclusion that the monetary policy stance does not affect the real interest rate level in the economy's ergodic distribution. Moreover, by analyzing the SSS of an economy without heterogeneity we could overlook the strength of the relationship between the inflation target and the real rate.

What drives the relationship between the inflation target and the real and nominal interest rates? To study this relationship, let us refer again to the Fisher equation. In the DSS of standard models, the real rate is the inverse of households' time discount parameter, whereas the level of inflation is a policy parameter. Given these two structural parameters, standard models determine the level of the nominal interest rate.

In the ZLB-HANK economy, by contrast, we have the deflationary bias triggered by the possibility of hitting the ZLB, which lowers the real rate and pushes inflation below its target, with both forces bringing down the nominal rate. While this deflationary bias also exists in the ZLB-RANK economy, it is stronger in the ZLB-HANK case because idiosyncratic uncertainty also lowers the real rate and makes hitting the ZLB more likely.

# 4 Persistent supply shocks and the natural rate

Finally, we reproduce the analysis in Nuno et al. (2024), in which the natural rate changes in response to persistent shocks such as tariffs or wars.

## 4.1 Model

**Households.** A continuum of identical households consume goods $c_t$, and supply labor $h_t$ to maximize the expected discounted utility:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\gamma}}{1-\gamma} - \frac{h_t^{1+\omega}}{1+\omega} \right) \right],$$

subject to the budget constraint:

$$p_t c_t + B_t \leq p_t w_t h_t + (1 + i_{t-1}) B_{t-1} + T_t,$$

where $B_t$ are holdings of a nominal bond which pays interest $1 + i_t$, $w_t$ is the real wage, $p_t$ is the price level, and $T_t$ are the profits from monopolistic producers.

We assume that the good is a basket of varieties indexed by $j$. Thus, the household also wants to choose the consumption of each variety $c_t(j)$ to minimize its expenditure. The outcome of the optimization problem is the optimal demand for $j$-th variety of good:

$$c_t(j) = \left( \frac{p_t(j)}{p_t} \right)^{-\epsilon} c_t. \tag{7}$$

where $\epsilon$ is the elasticity of substitution among varieties, $p_t(j)$ is the nominal price of the $j$-th variety and $p_t$ is the nominal price index for the basket.

**Firms.** There is a continuum of monopolistic firms, each of them producing a variety $j$. Each firm uses labor to produce the good according to the technology $y_t(j) = A_t h_t(j)$, where $A_t$ is the stochastic total factor productivity. There is a labor subsidy $\bar{\tau} = \frac{1}{\epsilon}$ to correct the distortions associated with monopolistic competition. Firms face temporary and persistent cost-push shocks, denoted by $\xi_t$ and $\eta_t$, respectively. Each firm's total cost is $(1 - \bar{\tau}) \eta_t w_t h_t(j)$. We define the labor wedge as

$$(1 + \tau_t) \equiv (1 - \bar{\tau}) \xi_t \eta_t, \tag{8}$$

which lumps together the labor subsidy and the cost-push shocks.

Each of these firms has monopoly power over their respective variety and takes the demand for its variety, $c_t(j)$, as given. We assume price stickiness *à la* Calvo: each retailer receives a random signal to adjust their prices with a probability $1-\theta$, allowing them to choose a new price $p_t(j)$ to maximize the stream of expected profits, that is:

$$\max_{P_t^*(j)} \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[ \frac{P_t^*(j)}{p_{t+k}} y_{t+k}(j) - \Psi\left(y_{t+k},(j)\right)\right]$$

subject to (7), where $\Psi\left(y_{t+k}(j)\right) \equiv \left(1+\tau_{t+k}\right) w_{t+k} \left(\frac{y_{t+k}(j)}{A_{t+k}}\right)$ are total costs, and $\Lambda_{t,t+k} \equiv \beta^k \frac{\lambda_{t+k}}{\lambda_t}$ is the stochastic discount factor for payments between periods $t$ and $t+k$. The rest of the firms maintain prices constant, that is, $p_{t+k}(j) = p_t(j)$.

We assume a symmetric equilibrium in which all firms are identical, and thus $p_t(j) = p_t$ holds. The optimal price is given by:

$$\frac{P_t^*}{p_t} = p_t^* = \mathcal{M} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} y_{t+k} \left(p_{t+k}/p_t\right)^{\epsilon} \Psi'\left(y_{t+k}\right)}{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} y_{t+k} \left(p_{t+k}/p_t\right)^{\epsilon-1}}, \tag{9}$$

where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$, and where $\Psi'\left(y_{t+k}(j)\right) = w_{t+k}\left(1+\tau_{t+k}\right)\left(A_{t+k}\right)^{-1}$.

**Central Bank.** The central bank sets the nominal interest rate on bonds. Bonds are in zero net supply $B_t = 0$. The central bank follows a Taylor rule of the form:

$$i_t = \frac{(1+\bar{\pi})}{\beta} - 1 + \psi\left(\pi_t - \bar{\pi}\right), \tag{10}$$

where $\bar{\pi}$ is the inflation target and $\psi$ is the slope of the Taylor rule as a function of inflation deviations.

**Market clearing conditions.** The market clearing conditions for goods are given by:

$$y_t\left(j\right) = c_t\left(j\right) + g_t,$$

where $g_t$ is a government spending shock. By aggregating, we obtain:

$$y_t = \int y_t(j)\, dj = \int (c_t(j) + g_t)\, dj = c_t + g_t.$$

Since all firms face the same probability $\theta$ of keeping prices fixed, the law of large numbers ensures that a fraction $\theta$ of firms will keep their prices fixed, while the remaining fraction $(1-\theta)$ will optimally reset their prices. As a result, the price level evolves according to:

$$p_t = \left\{\theta\,(p_{t-1})^{1-\epsilon} + (1-\theta)\,(P_t^*)^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}},$$

which implies

$$1 = \theta\,(1+\pi_t)^{\epsilon-1} + (1-\theta)\,(p_t^*)^{1-\epsilon}. \tag{11}$$

The market clearing condition for labor is given by:

$$h_t = \int h_t(j)\, dj = \left(\frac{y_t}{A_t}\right) \int \left(\frac{p_t(j)}{p_t}\right)^{-\epsilon} dj.$$

We define price dispersion as:

$$\Delta_t \equiv \int \left(\frac{p_t(j)}{p_t}\right)^{-\epsilon} dj = \theta\,(1+\pi_t)^{\epsilon}\,\Delta_{t-1} + (1-\theta)\,(p_t^*)^{-\epsilon},$$

such that the aggregate production function becomes:

$$y_t = A_t h_t \Delta_t^{-1}.$$

**Shocks and regimes.** We define the temporary and persistent shocks. We consider TFP, government spending, and cost-push shocks, each following an $AR(1)$ process in logs. First, we define:

$$g_t = \bar{g}\tilde{g}_t,$$

where $\bar{g}$ is a constant. Then, we have:

$$\log\left(A_t\right) = \left(1-\rho^A\right)\left(-\frac{\left(\sigma^A\right)^2}{2}\right) + \rho^A \log\left(A_{t-1}\right) + \varepsilon_t^A,$$

$$\log\left(\tilde{g}_t\right) = \left(1-\rho^g\right)\left(-\frac{\left(\sigma^g\right)^2}{2}\right) + \rho^g \log\left(\tilde{g}_{t-1}\right) + \varepsilon_t^g,$$

and:

$$\log(\xi_t) = \left(1-\rho^\tau\right)\left(-\frac{\left(\sigma^\tau\right)^2}{2}\right) + \rho^\tau \log(\xi_{t-1}) + \varepsilon_t^\tau,$$

where $\varepsilon_t^A \sim N\left(0, \sigma^A\right)$, $\varepsilon_t^g \sim N\left(0, \sigma^g\right)$, and $\varepsilon_t^\tau \sim N\left(0, \sigma^\tau\right)$.

We assume that the persistent cost-push shock, $\eta_t$, evolves according to a two-state Markov chain. We consider two regimes. In regime 1 "good times", the value of $\eta_t$ is one. In regime 2 "bad times", its value is $\bar{\eta} > 1$. This implies that the shock is only active during bad times. The transition probabilities are $p_{12}$ from regime 1 to 2:

$$p_{12} = \mathbb{P}\left(\eta_t = \bar{\eta} \mid \eta_{t-1} = 1\right), \tag{12}$$

and $p_{21}$ from 2 to 1

$$p_{21} = \mathbb{P}\left(\eta_t = 1 \mid \eta_{t-1} = \bar{\eta}\right). \tag{13}$$

All expectations are taken with respect to the AR(1) shocks and the persistent shock.

**Calibration and computational method.** The calibration is described in Appendix 5. The presence of the persistent shock complicates the model solution, as it requires solving the model globally with regime-switching dynamics. To address this issue, Nuno et al. (2024) employ an algorithm that extends the 'equilibrium nets"methodology of Azinovic et al. (2022) to accommodate a Markov-switching environment.

## 4.2 The natural rate and supply regimes

**Flexible-price equilibrium** We first consider the counterfactual equilibrium with flexible prices, that is, when $\theta = 0$. The optimal relative reset prices and price dispersion remain equal to one, $p_t^* = \Delta_t = 1$, reflecting that individual prices are always optimal. This setup creates a potential wedge between the marginal rate of substitution between consumption and labor and the marginal rate of transformation, as

$$h_t^{*\omega} c_t^{*\gamma} = \frac{A_t}{(1+\tau_t)\,\mathcal{M}} = \frac{A_t}{\eta_t \xi_t \,(1-\bar{\tau})\,\mathcal{M}} = \frac{A_t}{\eta_t \xi_t},$$

where, in the last equality, we apply the fact that the labor subsidy neutralizes the average markup. Here, $\eta_t \xi_t$ represents the cost-push shock, which has a mean of one in the normal times regime, but a mean of $\mathcal{M}$ in the persistent supply shocks regime. Consumption in this case satisfies the equation

$$c_t^* + g_t = y_t^* = A_t h_t^* = A_t^{\frac{1}{\omega}+1} (\eta_t \xi_t)^{-\frac{1}{\omega}} c_t^{*-\frac{\gamma}{\omega}}. \tag{14}$$

This can be rewritten as

$$\left(\frac{c_t^* + g_t}{A_t}\right)^{\omega} - \frac{A_t}{(\eta_t \xi_t) c_t^{*\gamma}} = 0. \tag{15}$$

Notice that consumption now depends on the cost-push shock. We define an SSS in this economy as a steady state where the innovations of the temporary and persistent shocks are zero and the temporary shocks remain at their mean values. This is an adaptation of the standard concept of SSS to the case of a Markov-switching model, which is instrumental in understanding model dynamics. Given that the persistent shock has two different values, the economy exhibits two SSSs, one in each regime.

**Natural rates** The real interest rate in the flexible-price economy satisfies the Euler equation

$$1 = \beta E_t \left[\frac{c_t^{*\gamma}}{c_{t+1}^{*\gamma}}\right] (1 + r_t^*).$$

If the economy is in regime 1, this equation implies

$$\frac{1}{\beta\left(1+r_t^*\right)} = {c_{1,t}^*}^{\gamma}\left(p_{12}E_t\left[\frac{1}{{c_{2,t+1}^*}^{\gamma}}\right] + \left(1-p_{12}\right)E_t\left[\frac{1}{{c_{1,t+1}^*}^{\gamma}}\right]\right),$$

where the notation $z_{n,t}$ denotes variable $z$ at time $t$ and regime $n = \{1,2\}$. The real rate in the SSS of regime 1 thus approximately satisfies

$$1 + r_{1,ss}^* \approx \frac{1}{\beta}\frac{{c_{2,ss}^*}^{\gamma}}{\left(p_{12}{c_{1,ss}^*}^{\gamma} + \left(1-p_{12}\right){c_{2,ss}^*}^{\gamma}\right)}, \tag{16}$$

reflecting that if the economy remains in regime 1, consumption remains at its SSS value $c_{1,ss}^*$, whereas if a regime change occurs in the next period, consumption jumps to the SSS in regime 2, $c_{2,ss}^*$. We denote the SSS value of the real rate in the flexible-price economy as the natural rate.

Each of these consumption levels is a solution to the SSS case of equation (14):

$$c_{n,ss}^* + \bar{g} = \left(\eta_n \xi_{n,ss}\right)^{-\frac{1}{\omega}}\left(c_{n,ss}^*\right)^{-\frac{\gamma}{\omega}},$$

where $\eta_n \xi_{n,ss}$ equals one in regime 1, and $\mathcal{M}$ in regime 2. The values are $c_{1,ss}^* = 0.9377$ and $c_{2,ss}^* = 0.8877$. Since $c_{2,ss}^* < c_{1,ss}^*$, the denominator on the right-hand side of equation (16) exceeds the numerator $(c_{2,ss}^*)^{\gamma}$. This implies that the natural rate in regime 1, $r_{1,ss}^*$, is *lower* than that in the efficient allocation, $1/\beta$. Conversely, the natural rate in regime 2, $r_{2,ss}^*$, is *higher* than that in the efficient allocation. In the calibration, these values are 0 percent and 2.7 percent, respectively, compared to 1 percent in the efficient allocation, as shown in the first column of Table 2.

The large differences in the natural rate as a result of the cost-push shock regime are driven by a *precautionary savings* motive by households. In normal times, households anticipate that, with a certain probability, the economy may shift to the other regime, where consumption will be lower. In anticipation of this event, they attempt to save more, but given the fixed supply of public debt —the only asset in this economy— their

| | Flex. prices | Taylor rule | Mod. Taylor rule |
|---|---|---|---|
| Inflation | | | |
| normal times | 0.0% | -0.9% | 0.0% |
| bad times | 0.0% | 1.6% | 0.0% |
| Output gap | | | |
| normal times | 0.0% | -0.1% | 0.0% |
| bad times | -5.5% | -5.4% | -5.5% |
| Real interest rates | | | |
| normal times | 0.0% | 0.1% | 0.0% |
| bad times | 2.7% | 2.6% | 2.7% |
| Nominal interest rates | | | |
| normal times | 0.0% | -0.8% | 0.0% |
| bad times | 2.7% | 4.2% | 2.7% |

Table 2: Stochastic steady state values.

increased demand for savings merely leads to a fall in the bond return, that is, in the natural rate. During bad times households are forced to reduce their savings to smooth consumption, which results in a higher natural rate.

## 4.3 Implications for monetary policy design

**Ergodic distribution and SSSs**. In the baseline case with nominal rigidities and a Taylor rule of the form (10), the central bank controls nominal interest rates to steer the economy towards an inflation level $\bar{\pi}$. Figure 8 shows the ergodic distribution in this case. It is obtained by simulating the economy for a large number of periods. The blue bars represent the share of the ergodic distribution that happens during the normal times regime, whereas the orange bars correspond to the periods in the persistent supply shock regime.

Several results emerge. First, the considered variables (inflation, output gap, nominal and real interest rates) exhibit *bimodality*: the distribution of realizations clusters around two distinct points. These two points correspond to the stochastic steady states (SSS) of each variable, as reported in the second column of Table 2.

Second, long-term inflation in this model is not zero. In the normal times regime it is negative (-0.9 percent) and in the bad times regime it is positive (1.6 percent). Such

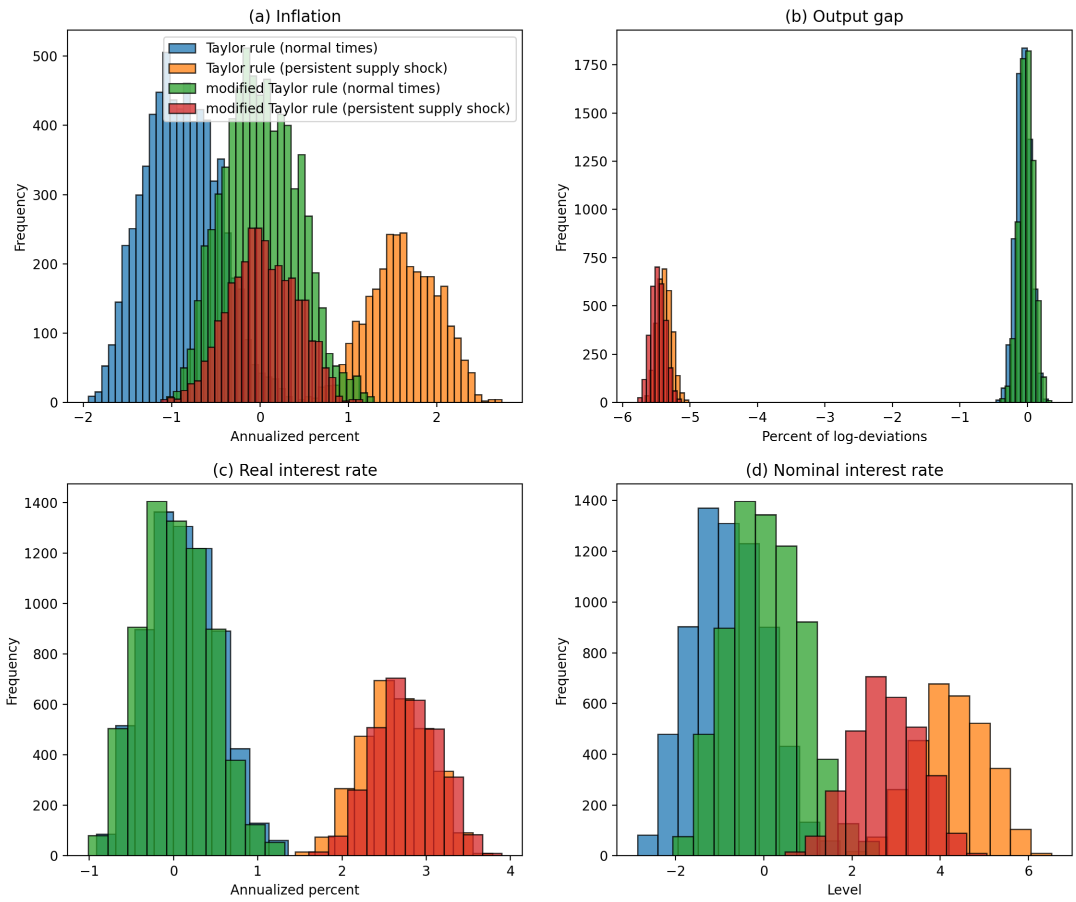


Figure 8: Ergodic distribution

*Note*: The figure displays the ergodic distribution in the model under a standard Taylor rule and a modified one. Colors distinguish the two regimes: blue denotes the samples corresponding to the standard Taylor rule in normal times, and orange in bad times. Green is the modified Taylor rule in normal times and red in bad times. The figure is produced by simulating the model for a large number of periods.

deviations from the central bank target $\bar{\pi} = 0$ are the result of the Taylor rule not targeting the adequate natural rate. If we evaluate the Taylor rule in a SSS $n = 1, 2$, we obtain

$$i_{n,ss} \simeq \left(\frac{1}{\beta} - 1\right) + \bar{\pi} + \psi\left(\pi_{n,sss} - \bar{\pi}\right) = \bar{r} + (1 - \psi)\,\bar{\pi} + \psi\pi_{n,sss},$$

where $\bar{r}$ is the real rate target of the central bank, which, under the Taylor rule (10), coincides with the real rate in the deterministic steady state of the efficient allocation,

$\bar{r} = \hat{r}$. Replacing the nominal rate using the Fisher equation $i_{n,ss} = r_{n,ss} + \pi_{n,sss}$, we get

$$\pi_{n,sss} \simeq \bar{\pi} + \frac{r_{n,ss} - \bar{r}}{\psi - 1}. \tag{17}$$

Notice how equation (17) can be seen as an SSS extension of equation (5) in Section 2 above. It illustrates how long-run inflation deviates from the central bank's target if the monetary policy rule targets an incorrect long-term real rate. In Section 2 the natural rate is endogenous to fiscal policy, whereas here it is the regime-switching nature of the cost-push shock that drives changes in the natural rate. In this model there is a significant gap between the natural rate in each regime and the central bank's target rate: in normal times, the central bank targets a natural rate that is too high, which tightens monetary policy excessively and explains why inflation is consistently below target: $\pi_{1,sss} \simeq 0.1\% - 1\% = -0.9\%$. Conversely, in bad times the central bank sets excessively low nominal rates, explaining why inflation is above target: $\pi_{2,sss} \simeq 2.6\% - 1\% = 1.6\%$.

Despite the central bank's failure to stabilize inflation in this economy, it meets its price stability mandate *on average*. Average inflation in the ergodic distribution is -0.1 percent, and the average real interest rate is 0.9 percent, satisfying equation (17) on average: $0.9\% - 1\% = -0.1\%$.

Third, real rates are slightly higher than natural rates in normal times and lower in the persistent shocks regime. This small divergence between the long-term real rates and the natural rates is due to the different values of SSS consumption. Compared to the flex-price allocation, consumption is lower in normal times and higher in bad times: the SSS values are $c_{1,ss} = 0.9368$ and $c_{2,ss} = 0.8883$. Following the logic of equation (16) once more, the jump in consumption between the two regimes becomes slightly narrower, and so does the gap in real interest rates.

The difference in consumption under the flex-price allocation is a consequence of the long-term impact of non-zero inflation on markups. Combining equations (9) and (11) in the steady state, we have

$$w_{n,ss} = A_t \left(\eta_n \xi_{n,ss}\right)^{-1} \left(\frac{1-\theta\left(1+\pi_{n,ss}\right)^{\epsilon-1}}{(1-\theta)}\right)^{\frac{1}{1-\epsilon}} \frac{1-\theta\beta\left(1+\pi_{n,ss}\right)^{\epsilon}}{1-\theta\beta\left(1+\pi_{n,ss}\right)^{\epsilon-1}}.$$

In the flex-price allocation, $\theta = 0$ and this expression simplifies to $w_{n,ss} = A_t \left(\eta_n \xi_{n,ss}\right)^{-1}$, such that in normal times wages coincide with those in the efficient allocation, $w_{1,ss} = A_t$, whereas in the bad times regime they are distorted $w_{1,ss} = A_t \left(\bar{\eta}\right)^{-1} < A_t$, which leads to lower labor and consumption in equilibrium. In the economy with nominal rigidities, long-term wages are also affected by long-term inflation. This distortion operates *against* the effect of persistent cost-push shocks. In the normal times regime, there is no distortion due to cost-push shocks. Still, negative inflation introduces an additional distortion on wages, which leads to slightly lower output and consumption. In the bad times regime, however, the distortion due to cost-push shocks is high, and positive inflation mitigates it to a limited extent, thus marginally increasing consumption. This can be confirmed by comparing the values of the output gap, -0.1% and -5.4%, with those under flexible prices, 0% and -5.5%. In both regimes, non-zero inflation increases price dispersion, but this effect is second order compared to that on the average markup.

**Modified Taylor rule** Finally, we consider an alternative monetary policy rule that is regime-contingent. The new Taylor rule is

$$i_t = \bar{r}_t + \psi\left(\pi_t - \bar{\pi}\right), \tag{18}$$

where the Taylor rule intercept equals the natural rate in each regime

$$\bar{r}_t = r_n^*, \text{ if the regime at time } t \text{ is } n.$$

Here $r_n^*$ is the natural rate in regime $n$, that is, the SSS real rate in the counterfactual flex-price allocation. In this case, it is easy to see that equation (17) is compatible with a zero inflation target $\bar{\pi} = 0$. The third column in Table 2 confirms this: inflation is zero in both SSSs and real rates and output gaps coincide with those in the flex-price allocation. The red and green bars in Figure 8 display the ergodic distribution under this modified Taylor rule. They are again centered around the SSS values. In particular, the inflation distribution is centered around zero inflation. The variances of the different variables are similar under the original and the modified Taylor rules.

The policy prescription is clear: in the presence of persistent supply shocks the central bank should endogenously adapt its interest rate target to track the natural rate, which becomes a regime-contingent object.

# 5 Conclusion

This paper summarizes recent research in macroeconomics challenging the traditional view that the natural rate only depends on slow-moving structural factors.[20] We show that straightforward extensions to the New Keynesian model, such as considering household-heterogeneity or persistent supply shocks, open the door to fiscal and monetary policy affecting the natural rate, and how the latter may jump depending on the macroeconomic regime. A common feature of these extensions is the important role of precautionary motives, at the micro and/or macro level.

The possibility that the natural rate may exhibit medium- and high-frequency dynamics makes central bankers' work more complex. If fiscal policy and persistent shocks, such as tariffs, shift the natural rate, the monetary policy stance, including the long end of the risk-free yield curve, can change abruptly due to factors outside the central bank's control. The optimal reaction of the central bank to these changes should be a key research topic in the years to come.[21]

[20] Complementary empirical work challenging this view can be found in Rogoff et al. (2024).

[21] Some early results have already been put forward by Daudignon and Tristani (2023) and Nuno et al. (2024).

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# Appendix

## A. Calibrations

### A.1. Calibration of the model in Section 2

We assume log utility over consumption: $u(c) = \log(c)$. The disutility over hours is parameterized using a function with a constant Frisch elasticity: $v(n) = \nu_\varphi n^{1+\frac{1}{\varphi}}/(1+\frac{1}{\varphi})$. We set the Frisch elasticity $\varphi$ to 0.5. The preference shifter $\nu_\varphi = 0.791$ is calibrated so that, given all other parameters, total employment is 1 in the steady state. This is an immaterial normalization. We calibrate the discount factor to match a real interest rate of 1% annually. This implies a quarterly discount factor of $\beta = 0.992$.

The persistence and standard deviation of income shocks are taken from the estimates of the US wage process in Floden and Lindé (2001). We set the persistence of the income process to match a persistence of 0.91 yearly and the standard deviation of innovations to match the standard deviation of log gross earnings of 0.92. We first convert these values to quarterly frequency and then approximate the income process with a Markov chain with 11 discrete states calculated as in Rouwenhorst (1995).[22]

We discretize the asset space using a double-exponential transformation of a uniformly spaced grid using 500 grid points, with a minimum asset level of $\underline{a} = 0$ (the borrowing limit) and a maximum asset level of $\overline{a} = 150$. We solve the household problem using the endogenous grid method (Carroll, 2006; Barillas and Fernandez-Villaverde, 2007).

We normalize the steady-state quarterly output and total factor productivity $\Theta$ to one. This implies that total hours equal output in the steady state. We set the value of elasticity between labor tasks $\epsilon_w$ to 10, which is a standard value in the literature (see, for example, Wolf, 2021). We take the slope of the wage Phillips curve $\kappa_w = 0.1$ from Aggarwal et al. (2023).

We assume a conventional target of government consumption of 20% of GDP, which

[22] The quarterly persistence is calculated as $\rho_Q = 0.92^{1/4} \approx 0.977$ and the quarterly standard deviation of innovations as $\sqrt{(0.91^2/(\sum_{t=0}^{3}\rho_Q^{2t})} \approx 0.476$.

is close to the US data and is also the number used by Auclert et al. (2018). Because we have normalized quarterly output to one, this implies that $\overline{G} = 0.2$. We take as a benchmark the case in which the annual real interest rate is 1%, and public debt stands at 70% of annual GDP. In this quarterly model, the stock of public debt at which the real interest rate is at its steady-state value is, therefore, $\overline{B} = 4 \times 0.7 = 2.8$. The values of the tax rate and net transfers are chosen to balance the budget in the steady state and ensure that public debt remains constant, i.e. that total tax revenue matches government spending plus net transfers and interest payments on public debt. The OECD reports a tax-to-GDP ratio of 0.277 for the United States in 2022. We set transfers to 7% of annual GDP, so that $\mathcal{T}_{ss} = G_{ss} + T_{ss} + r_{ss} \times B_{ss} = 0.2 + 0.07 + 0.01/4 \times 2.8 = 0.277$. Because $\mathcal{T}_{ss} = \tau Y_{ss}$ and $Y_{ss} = 1$, this implies that $\tau = 0.277$. Finally, for the baseline case, we set the parameter $\phi_G$, which governs how quickly $G_t$ responds to deviations from the debt target, to 0.1.

We parameterize the Taylor rule to achieve an inflation target of 2% annually and set the Taylor rule coefficient to 1.25.

### A.2. Calibration of the model in Section 3

We set the gross inflation target of the central bank to $\bar{\pi} = \exp(0.02/4)$, so that the annual inflation target is 2%. For the Taylor rule, we set the sensitivity of the nominal rate to output deviations from the DSS to $\phi_y = 0.1$, and that of changes in inflation to $\phi_\pi = 2.5$. We set $\beta = 0.99577$ such that the real interest rate in the DSS of the model equals 1.5%.

We calibrate the preference shifter by making the model consistent with the 10% frequency of ZLB episodes observed in the US economy in the post-war period (Coibion et al., 2016), by first setting $\rho_\xi = 0.6$, in line with the parameterization in Bianchi et al. (2021), and then setting $\omega_\xi = 0.01225$.

For the labor market, we set the elasticity across differentiated labor services to $\varepsilon = 11$, such that the wage markup is 10%. The slope of the Phillips curve is $\kappa_w = 0.11$.

Regarding idiosyncratic risk, we first set $\underline{a} = -0.58$, which equals about two months' average wages. We set the persistence of the income process and the standard deviation of innovations to match the standard deviation of log gross earnings, so that the model reproduces a share of borrowers of 33% (Kaplan et al., 2014), and an average MPC of around 10%, which is at the lower end of the estimates provided by the literature (Johnson et al., 2006, Parker et al., 2013, and Broda and Parker, 2014).

We calibrate the remaining parameters for households as follows. We assume log utility over consumption: $u(c) = \log(c)$ and set the Frisch elasticity to $\varphi = 1$. Finally, we normalize the disutility of labor to $\nu_\varphi = 0.8696$ such that the aggregate value of the efficiency units of hours equals one in the DSS.

Regarding the fiscal authority, we follow the one-asset calibration strategy of McKay et al. (2016) and set $\widetilde{B} = 25\%$ of annual GDP, in line with the estimate of liquid wealth in the US economy derived by Kaplan et al. (2018).

### A.3. Calibration of the model in Section 4

The model outlined in Section 4 is calibrated at a quarterly frequency, and the parameters are reported in Table 3. The calibration relies on standard values from the literature as much as possible. Regarding preferences, the quarterly discount factor is 0.9975, implying a real interest rate of 1% in the deterministic steady state. The elasticity of substitution across products is $\epsilon = 7$, resulting in a frictionless net markup of 1/6. The inverse of the intertemporal elasticity of substitution $\gamma$ is set to 2, and the inverse of the Frisch elasticity $\omega$ is set to 1. The long-run productivity level $A$ is normalized to one, and the government spending constant $\bar{g}$ is set to 20%.

The inflation coefficient of the Taylor rule $\psi$ is set to 2, and the inflation target $\bar{\pi}$ is zero.

The TFP, government spending and cost-push shock parameters are taken from Coibion et al. (2012). The value of the persistent cost-push shock is set to $\bar{\eta} = \mathcal{M}$, so that it fully offsets the optimal labor subsidy. The average duration of regime 1 ("normal times") is 48

| Parameter | | Value |
|---|---|---|
| Long-run productivity level | $A$ | 1 |
| Inverse Frisch elasticity | $\omega$ | 1 |
| Inverse of intertemporal elasticity of substitution | $\gamma$ | 2 |
| Discount factor | $\beta$ | 0.9975 |
| Elasticity of substitution among varieties | $\epsilon$ | 7 |
| Government spending constant | $\bar{g}$ | 0.2 |
| Calvo constant | $\theta$ | 0.75 |
| Taylor rule slope | $\psi$ | 2 |
| Inflation target | $\bar{\pi}$ | 0 |
| Labor subsidy | $\bar{\tau}$ | $\frac{1}{\epsilon}$ |
| | | |
| Mean of cost-push shock during persistent supply shock | $\bar{\eta}$ | $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$ |
| Transition probability from normal to negative supply times | $p_{12}$ | 1/48 |
| Transition probability from negative supply to normal times | $p_{21}$ | 1/24 |
| Persistence of TFP shock | $\rho^A$ | 0.99 |
| Persistence of cost-push shock | $\rho^\tau$ | 0.90 |
| Persistence of government spending shock | $\rho^g$ | 0.97 |
| Standard deviation of TFP shock | $\sigma^A$ | 0.009 |
| Standard deviation of cost-push shock | $\sigma^\tau$ | 0.0014 |
| Standard deviation of government spending shock | $\sigma^g$ | 0.0052 |

Table 3: Key parameters of the model.

quarters (12 years), to capture a period encompassing a business cycle, giving $p_{12} = 1/48$, while the average duration of regime 2 ("persistent supply shock") is set to half of that of normal times, 24 quarters (6 years), meaning that $p_{21} = 1/24$.