Continuous-time Methods in Macroeconomics

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Lectures: Monday-Friday 9-10:30, 11-12:30.

Course outline: The aim of this course is to introduce a number of analytical and computational tools that are useful for macroeconomics, and in particular for the study of models in which the relevant state variable is a distribution. Rather than presenting an in-depth technical derivation of the methods my intention is to focus on relevant applications.

At the beginning of the course we will introduce dynamic programming (DP) in continuous time, as well as some key theoretical results in stochastic calculus. Then we will present the main numerical techniques to solve continuous-time DP problems, including finite difference and deep learning. These techniques have proven to very useful in the solution of models with heterogeneous agents, which constitute the core of the course. We will first analyze the workhorse of this literature, the Aiyagari-Bewley-Hugett model, which has been successfully applied to the study of diverse issues such as why income and wealth are so unequally distributed or which are the main transmission channels of monetary policy. We will move then to discuss how to introduce aggregate shocks in this environment. Finally, we will review the latest advances in the computation of optimal policies in models in which the state variable is a distribution.

Computer codes: I provide computer codes to simulate all the examples presented in the course. Students are expected to run them as homework.

Readings:

1. Dynamic programming and stochastic calculus.

Van Handel, R. (2007). Lectures notes on *Stochastic Calculus, Filtering, and Stochastic Control*, Chapters 5, 8. https://web.math.princeton.edu/∼rvan/acm217/ACM217.pdf.

Björk, T. (2009). *Arbitrage Theory in Continuous Time*, Oxford University Press. Chapters 4-7, 19.

Øksendal, B. (2007). *Stochastic Differential Equations: An Introduction with Applications*. Springer. Chapters 3-5, 11.

Brunnermeier, M. K, T. Eisenbach, and Y. Sannikov (2013). "Macroeconomics With Financial Frictions: A Survey," *Advances In Economics And Econometrics*. Cambridge University Press.

Merton, R. C. (1969). "Lifetime Portfolio Selection under Uncertainty: the Continuous-Time Case," *The Review of Economics and Statistics* 51 (3), pp. 247–257.

2. Numerical techniques for dynamic programming problems.

Parra-Alvarez. J.-C. (2018). "A comparison of numerical methods for the solution of continuous-time DSGE models," Macroeconomic Dynamics, 22(6), pp. 1555-1583.

Barles, G. and P. E. Souganidis (1991). "Convergence of Approximation Schemes for Fully Nonlinear Second Order Equations," *J. Asymptotic Analysis*, 4, pp. 271-283.

Achdou, Y., J.-M. Lasry, P.-L. Lions and B. Moll (2017)."Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach ," mimeo.

Fleming, W. H. and H. M. Soner (2006). *Controlled Markov Processes and Viscosity Solutions*, Springer. Chapter 9.

Nuño, G. and C. Thomas (2019). "Monetary Policy and Sovereign Debt Sustainability," Bank of Spain Working Paper.

Goodfellow, I., Bengio, Y., and Courville, A. (2016). *Deep Learning*. MIT Press.

Duarte, V. (2018). "Machine Learning for Continuous-Time Finance,"mimeo.

Sirignano, J. and K. Spiliopoulos (2018). "DGM: A deep learning algorithm for solving partial differential equations," *Journal of Computational Physics*.

Fernández-Villaverde, J,. G. Nuño, G. Sorg-Langhans, and M. Vogler (2019), "Deep Learning for Large State Space Models," mimeo.

3. Heterogeneous agent models in continuous time.

Aiyagari, R., (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, 109 (3), pp. 659-84.

Bewley, T. (1986). "Stationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers." In *Contributions to Mathematical Economics in Honor of Gerard Debreu*, ed. Werner Hildenbrand and Andreu Mas-Collel. Amsterdam: NorthHolland

Huggett, M. (1993), "The Risk-free rate in Heterogeneous-agent Incomplete-insurance Economies," *Journal of economic Dynamics and Control*, 17 (5-6), pp. 953-969.

Heathcote J., K. Storesletten and G. L. Violante (2009). "Quantitative Macroeconomics with Heterogeneous Households," *Annual Review of Economics*, 1, pp. 319-54.

Achdou, Y., J.-M. Lasry, P.-L. Lions and B. Moll (2017). "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach ," mimeo.

Kaplan, G., B. Moll and G. Violante (2018). "Monetary Policy According to HANK," *American Economic Review*, 108(3), pp. 697–743.

4. Heterogeneous agent models with aggregate shocks.

Algan, Y., Allais, O., Den Haan, W. J., and Rendahl, P. (2014). *Solving and simulating models with heterogeneous agents and aggregate uncertainty.* In Schmedders, K. and Judd, K. L., editors, Handbook of Computational Economics, 3, pp. 277–324.

Ahn, S., G. Kaplan, B. Moll, T. Winberry and C. Wolf (2018). "When Inequality Matters for Macro and Macro Matters for Inequality," *NBER Macroeconomics Annual.*

Boppart, Krusell and Mitman (2018). "Exploiting MIT Shocks in Heterogeneous-Agent Economies: The Impulse Response as a Numerical Derivative", *Journal of Economic Dynamics and Control*, 89, pp. 68-92.

Krusell, P. and Smith, A. (1998). "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), pp. 867–896.

Fernández-Villaverde, J., S. Hurtado and G. Nuño (2019). "Financial Frictions and the Wealth Distribution," NBER Working Paper No. 26302.

5. Optimal policies with heterogeneous agents.

Luenberger D. (1969). *Optimization by Vector Space Methods*, Ed. Wiley-Interscience, NJ.

Lucas, R. and B. Moll (2014). "Knowledge Growth and the Allocation of Time," *Journal of Political Economy*, 122 (1), pp. 1-51.

Nuño, G. and B. Moll (2018). "Social Optima in economies with Heterogeneous Agents," *Review of Economic Dynamics*, 28, pp. 150-180.

Dávila, J., J. H. Hong, P. Krusell and J. V. Ríos-Rull (2012). "Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks," *Econometrica*, 80(6), pp. 2431-2467.

Bigio, S., G. Nuno and J. Passadore (2019). "A Framework for Debt-Maturity Management," NBER Working Paper No. 25808.

Nuño, G. and C. Thomas (2019). "Optimal Monetary Policy with Heterogeneous Agents," Bank of Spain Working Paper.

Bigio. S. and Y. Sannikov (2019). "A Model of Credit, Money, Interest, and Prices,"mimeo.