## Firm Heterogeneity, Capital Misallocation and Optimal Monetary Policy

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How does capital misallocation affect the optimal conduct of monetary policy?

- Capital misallocation depends on firms investment decisions
- ▶ Monetary policy affects investment decisions → Capital misallocation.
- Especially relevant in times of large hikes in interest rates.
- How does it affect the optimal conduct of monetary policy?

This paper: heterogeneous firms, financial frictions, and nominal rigidities

- Capital misallocation channel of monetary policy: Expansionary monetary policy reduces capital misallocation => increases TFP
  - by increases share of investment by high-MRPK firms
  - Supported by empirical analysis based on Spanish firm-level micro data

# This paper: heterogeneous firms, financial frictions, and nominal rigidities

- Capital misallocation channel of monetary policy: Expansionary monetary policy reduces capital misallocation 

   increases TFP
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#### Optimal monetary policy:

- Misallocation creates a *time inconsistent* motive to temporarily expand the economy.
- Timeless response to shock HH time preference, TFP or financial shocks: price stability ("divine coincidence" holds)...
- ... but at the ZLB: low for even longer.

## Model

## The model in a nutshell



- Heterogeneity in entrepreneurs' net worth ( $a_t$ ) and productivity (follows OU-diffusion process,  $dlog(z_t) = -(1/\theta) \log z \, dt + \sigma \sqrt{1/\theta} \, dW$ ;).
- Firms produce the input good using labor  $(I_t)$  and capital  $(k_t)$  (CRS).
- Entrepreneurs can borrow capital  $b_t = k_t a_t$ , subject to a borrowing constraint  $k_t \le \gamma a_t$ .

#### Entrepreneurs maximize profits...

Entrepreneurs are household's members (as in Gertler & Karadi, 2011, unlike Moll, 2014).

• Maximize profits;  $\Phi_t(z_t, a_t) = \max_{k_t, l_t} \{ m_t f_t(z_t, k_t, l_t) - w_t l_t - R_t k_t \}$ ; s.t.  $k_t \leq \gamma a_t$ 

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If z < z<sub>t</sub><sup>\*</sup>, optimal size is k<sub>t</sub>(z, a) = k<sub>t</sub><sup>\*</sup>(z) = 0 → Entrepreneur is unconstrained
 She lends her net worth to other entrepreneurs.

▶ If  $z \ge z_t^*$ , operate at maximum capacity  $k_t(z, a) = \gamma a \rightarrow$  Entrepreneur is constrained

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$$V_0(z, a) = \max_{a_t, d_t \ge 0} \mathbb{E}_0 \int_0^\infty e^{-\int_0^t (r_s + \eta) ds} \begin{pmatrix} \mathsf{liquidation value} \\ d_t + \eta q_t a_t \end{pmatrix} dt$$

s.t.



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- Entrepreneurs optimally never distribute dividends until liquidation.
  - Intuition: return of funds inside the firm is always at least the real rate  $\left(\frac{R_t \delta q_t}{q_t}\right)$ , and the liquidation value of the firm is all its net worth.

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- New entrepreneurs enter replacing exiting ones.
  - Inherit the same firm (same productivity); start with lower net worth  $\psi q_t a_t$ ,  $0 < \psi < 1$ .

#### Distribution in net worth shares

The evolution of the joint distribution of net worth and productivity  $g_t(z, a)$  is given by the KFE:

$$\frac{\partial g_{t}(z,a)}{\partial t} = \underbrace{-\frac{\partial}{\partial a}[g_{t}(z,a)s_{t}(z)a]}_{\text{Entrepreneurs' savings}} \underbrace{-\frac{\partial}{\partial z}[g_{t}(z,a)\mu(z)] + \frac{1}{2}\frac{\partial^{2}}{\partial z^{2}}[g_{t}(z,a)\sigma^{2}(z))]}_{\text{idiosyncratic productivity shocks}}$$

• Only need the distribution of net worth shares  $\omega_t(z) = \frac{1}{A_t} \int_0^\infty ag_t(z, a) da$ .

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- Only need the distribution of net worth shares  $\omega_t(z) = \frac{1}{A_t} \int_0^\infty ag_t(z, a) da$ .
- Model is isomorphic to standard RANK with endogenous TFP  $Z_t$ ,

$$Z_t = \left(\mathbb{E}_{\omega_t(\mathbf{z})}\left[z \mid z > \mathbf{z}_t^*\right]\right)^{\alpha}.$$

#### Misallocation leads to endogenous TFP



#### Capital misallocation channel of monetary policy

$$\tilde{Z}_t = \left( \mathbb{E}_{\omega_{t}(\mathbf{z})} \left[ z \mid z > \mathbf{z}_t^* \right] \right)^{\alpha}$$

Net worth shares,  $\omega(z)$ 

Changes in the distribution of net-worth across firms

$$ilde{\Phi}_t(z) = rac{\gamma}{q_t} \left( z_t arphi_t - oldsymbol{R}_t 
ight) = rac{\gamma arphi_t}{q_t} (z_t - z_t^*)$$

Changes in the share of constrained firms

$$z_t^* \varphi_t = R_t$$





Productivity, z

### New Keynesian Block

- Representative Household More
- Capital good producer
- Retailers More
  - New Keynesian Phillips Curve More

► Final good producer ► More



## **Equilibrium Prices and Misallocation**

#### Calibration

#### Table 1: Calibration

	Parameter	Value	$\mathbf{Source}/\mathbf{target}$
$\psi$	Fraction firms' assets at entry	0.1	Capital of firms younger than 1 year / All firms capital
$\eta$	Firms' death rate	0.1	Average exit rate
$\gamma$	Borrowing constraint parameter	1.56	Debt / Total Assets firms
$\varsigma_z$	Mean reverting parameter	0.19	Estimate based on firm level data
$\sigma_z$	Volatility of the shock	0.73	Estimate based on firm level data
$\rho^h$	Household's discount factor	0.01	1%
$\alpha$	Capital share in production function	0.35	Gopinath et al. 2017
$\delta$	Capital depreciation rate	0.06	Gopinath et al. 2017
ζ	Intertemporal elasticity of substitution HH	1	Log utility in consumption
θ	Inverse Frisch Elasticity	1	Kaplan et al. (2018)
Υ	Constant in disutility of labor	0.71	Normalization $L = 1$
$\phi^{k}$	Capital adjustment costs	8	VAR evidence Christiano et al. $(2016)$
$\epsilon$	Elasticity of substitution retail goods	10	Mark-up of 11%
$\theta$	Price adjustment costs	100	Slope of Phillips curve of $0.1~{\rm as}$ in Kaplan et al. (2018)
$\bar{\pi}$	Inflation target	0	Standard
$\phi$	Slope Taylor rule	1.5	Standard
v	Persistence Taylor rule	0.2	Standard

#### Calibration

**MRPK** distribution



#### Natural rates, real rates, and misallocation



Price decomposition

## **Optimal Monetary Policy**

#### Central Bank's Ramsey problem

$$\max_{\{\omega_t(z), \operatorname{Prices}_t, \operatorname{Quantities}_t\}_{t \in [0, \infty)}} \mathbb{E}_0 \int_0^\infty e^{-\rho^h t} u(C_t, L_t) dt$$

subject to private equilibrium conditions  $\forall t \in [0,\infty)$  and initial conditions

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- Need to keep track of the whole distribution of firms  $\omega_t(z)$
- We propose a new algorithm to solve for Ramsey optimal policies with heterogeneous agents.
  - Discretize the continuous time and continuous-space problem and solve non-linearly for the optimal monetary policy in the sequence space using symbolic differentiation and Newton methods. More

## Optimal Ramsey policy: a new time inconsistency



- Complete Markets economy (CM): zero inflation is optimal (SS is first-best due to subsidy undoing mark-up distortion) CE vs Baseline
- Baseline economy: surprise inflation is optimal since it reduces capital misallocation
- Same response as expansionary monetary policy shock

## Timeless optimal response to HH time preference shock



Divine coincidence holds as in the complete markets economy

Similar to other shocks (e.g. TFP, financial shock)

## Timeless optimal response with ZLB: low for even longer



- If planner were not constrained by ZLB (light blue), she would decrease further nominal rates (blue) as compared to the ZLB case.
- Heterogeneity and financial frictions calls for '*low for longer*' compared to the complete markets case (yellow).

## **Empirical Evidence**

1. After a monetary policy expansion, high-MRPK firms increase their investment relatively more

- Data: CBI- yearly balance sheet and cash flow data for the quasi-universe of Spanish firms
- $\varepsilon_t^{MP}$  monetary policy shocks from Jarociński and Karadi (2020)
- Local projection

$$\log k_{j,t} - \log k_{j,t-1} = \beta_0 + \beta_1 \log (MRPK_{j,t-1}) + \beta_2 \log (MRPK_{j,t-1}) \varepsilon_t^{MP} + \gamma_{st} + u_{j,t},$$

Response of investment to an expansionary monetary policy shock as a function of initial MRPK



Notes: The figure displays the average effect of an 1 p.p. expansionary monetary policy shock on the growth rate of the capital stock in the year after the shock in p.p.  $-100 \times [\log k_{j,1} - \log k_{j,0})$  - as a function of the firms' log MRPK before the shock  $\log(MRPK_{j,0})$ . For the model (orange), the relationship is calculated analytically. See Appendix for more detail. Estimating the regression on simulated data would recover a linear approximation of it. We compare the model prediction to the estimated relationship (black). The shaded areas mark the 90. S5 and 99% confidence intervals.

Response of firm-level investment to an expansionary

monetary policy shock

	(1)	(2)
	$\Delta logk_{j,t-1,t}$	$\Delta logk_{j,t-1,t}$
$\varepsilon_t log(MRPK_{j,t-1})$	0.0286***	0.0470***
<b>C</b> .	(0.01)	(0.02)
$\varepsilon_t$		(0.02)
Obs	3,692,188	3,692,188
$R^2$	0.02	0.01
$\gamma_{st}$	Yes	No
$\gamma_s$	No	Yes

Notes: Column (1) reports the differential effect ( $\beta_2$ ) estimated from regression including sector-year fixed effects. Column (2) reports the estimated differential effect ( $\beta_2$ ) and average effect ( $\beta_3$ ), including sector fixed effects, aggregate controls (lagged GDP growth, inflation and unemployment). Standard errors clustered at the sector-year level.

2. After a monetary policy expansion, misallocation is reduced

Isolate contribution of capital reallocation to TFP by computing the dynamics of weighted average MRPK with constant initial firm-level MRPK

$$\textit{WAM}_{t, au} = \sum_{j=0}^{J}\textit{MRPK}_{t}^{j} rac{\textit{k}_{t+ au}^{j}}{\textit{K}_{t+ au}}$$

Local projection (at sector level)

$$\Delta \log WAM_{t,\tau,s} = \alpha_{s,\tau} + \beta_{\tau} \varepsilon_t^{MP} + u_{s,t,\tau}$$



#### Conclusions

Model of heterogeneous firms, financial frictions and monetary policy

Including a new algorithm to solve and compute optimal policy

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#### Optimal Monetary Policy:

- New source of inflationary time inconsistency: undoing financial frictions.
- Divine coincidence holds when facing households' discount factor shock (timeless)
  - \* Zero-Lower Bound: Low for even longer.

Thank you!

Appendix

#### Representative household



Standard consumption-labor-savings choice

$$\max_{C_t, L_t, D_t, B_t^N} \mathbb{E}_0 \int_0^\infty e^{-\rho^h t} u(C_t, L_t) dt$$

$$\dot{D}_t q_t + \dot{B}_t^N + C_t = (R_t - \delta q_t) D_t + (i_t - \pi_t) B_t^N + w_t L_t + T_t$$

 $\succ$   $C_t$ : consumption

 $D_t$ : capital holdings

 $B_t^N$  holdings of nominal bonds (zero net supply)

- $\blacktriangleright$  *L<sub>t</sub>*: labor supply
- *i*t: nominal interest rate
- T<sub>t</sub> : profits of retailers, capital good producer and net dividends from firms

### Capital good producer

Produces capital and sells it to the household and the firms at price  $q_t$ 



$$\max_{\iota_{t},K_{t}} \mathbb{E}_{0} \int_{0}^{\infty} e^{-\int_{0}^{t} r_{s} ds} \left( q_{t} \iota_{t} - \iota_{t} - \Xi(\iota_{t}) \right) K_{t} dt.$$

$$s.t. \quad \underbrace{\dot{K}_{t} = (\iota_{t} - \delta) K_{t}}_{\text{LOM of } K_{t}}.$$

ι<sub>t</sub>: investment rate,

►  $\equiv (\iota_t) = \frac{\phi^k}{2} (\iota_t - \delta)^2$ : quadratic adjustment costs.

## New Keynesian block

► Final good producers aggregate varieties j ∈ [0, 1]. Cost minimization implies demand for variety j is given by

$$y_{j,t}(p_{j,t}) = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t, \text{ where } \underbrace{P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}}_{\text{Aga, Price index}}.$$

Retailers maximize

$$\max_{P_{j,t}} \int_{0}^{\infty} e^{-\int_{0}^{t} r_{s} ds} \left\{ \underbrace{\left(\frac{p_{j,t}}{P_{t}} - m_{t}\right)}_{\text{Mark-up}} \left(\frac{p_{j,t}}{P_{t}}\right)^{-\varepsilon} Y_{t} - \frac{\theta}{2} \left(\frac{\dot{p}_{j,t}}{p_{j,t}}\right)^{2} Y_{t} \right\} dt$$

- ε: elasticity of substitution across goods
   ε > 0.
- $\triangleright$   $p_{j,t}$ : price of variety j.

•  $\theta$ : price adjustment cost parameter.

#### New Keynesian block

Back

The symmetric solution to the pricing problem yields the New Keynesian Phillips curve

$$\left(r_t - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\varepsilon}{\theta}\left(m_t - m^*\right) + \dot{\pi}_t, \quad m^* = \frac{\varepsilon - 1}{\varepsilon},$$

•  $\pi_t = \frac{\dot{P}}{P_t}$  is inflation,

- *m<sub>t</sub>* are relative prices of intermediate good (inverse mark-ups of retailers),
- *m*<sup>\*</sup> is the optimal inverse mark-up,
- Real rates are defined as  $r_t \equiv \frac{R_t \delta q_t + \dot{q}_t}{q_t}$ .

#### **Central Bank**

#### Back

- The central bank controls nominal interest rates it on nominal bonds held by households.
- Positive analysis: central bank sets the nominal rate according to a Taylor rule of the form

$$di = -\upsilon \left( i_t - \left( \rho^h + \phi \left( \pi_t - \bar{\pi} \right) + \bar{\pi} \right) \right) dt, \tag{1}$$

where  $\bar{\pi}$  is the inflation target,  $\phi$  is the sensitivity to inflation deviations and v is a parameter related to the persistence.

$$\begin{split} \mathbf{w}_t &= (1-\alpha) m_t Z_t K_t^{\alpha} L_t^{-\alpha}, \\ R_t &= \alpha m_t Z_t K_t^{\alpha-1} L_t^{1-\alpha} \frac{Z_t^*}{\mathbb{E} \left[ z \mid z > Z_t^* \right]}, \\ & \frac{\dot{A}_t}{A_t} &= \frac{1}{q_t} \left[ \gamma (1 - \Omega(Z_t^*)) \left( \alpha m_t Z_t K_t^{\alpha-1} L_t^{1-\alpha} - R_t \right) + R_t - \delta q_t - q_t (1-\psi) \eta ) \right]. \end{split}$$

#### Permanent decrease in natural rates decreases TFP



### Sketch of solution algorithm

Back

Discretize the time space (Δt); and the state space (Δz) into J grid points using finite differences (Achdou et al, 2017):

system of 2J equations and 2J unknowns for the HJB and the KFE equation (we don't have a HJB).

$$\begin{pmatrix} \frac{1}{\Delta t} \left( \mathbf{v}^{n+1} - \mathbf{v}^{n} \right) + \rho \mathbf{v}^{n+1} &= \mathbf{u}^{n+1} + \mathbf{A}^{n+1} \mathbf{v}^{n+1} \\ \frac{\mathbf{g}^{n+1} - \mathbf{g}^{n}}{\Delta t} = \left( \mathbf{A}^{n+1} \right)^{T} \mathbf{g}^{n+1} \end{pmatrix}$$

- set of X equilibrium conditions (MC, FOCs of representative agents)
- Compute the planner's optimality conditions on discretized problem : (2J + X) + (2J + X + 1) equations using symbolic differentiation
- Solve the transitional dynamics up to horizon T using a Newton algorithm to solve a large equation set of [(2J + X) + (2J + X + 1)] T equations (cf. Auclert et al., 2020)

#### **RANK vs HANK**

#### Back

#### RANK

- ▶ All capital is owned by HH  $D_t = K_t$
- No financial frictions.
- TFP is exogenous Z = 1

#### HANK

- Capital is owned by HH and entrepreneurs:  $D_t + A_t = K_t$
- Financial frictions:  $k_t \leq \gamma a_t$
- ► TFP is endogenous  $Z = (\mathbb{E}_t [z \mid z > z_t^*])^{\alpha}$

Introduce subsidies in both economies, such that the SS mark-up distortion is undone.

#### Price decomposition

Back



Notes: The figure decomposes the effect of a monetary policy shock on TFP (bold blue line) into the effect of the individual factor price changes. This is done by computing how TFP would have evolved if all prices but one would have remained at steady state.

#### Robustness

	$^{(1)}_{\Delta logk}$	(2) $\Delta logk$	(3) $\Delta logk$	$^{(4)}_{\Delta logk}$	(5) $\Delta logk$	(6) $\Delta logk$	(7) $\Delta logk$	(8) $\Delta logk$	$^{(9)}_{\Delta logk}$	(10) $\Delta logk$	(11) $\Delta lognop$
$\varepsilon*mrpk$	$0.0286^{***}$ (0.01)	$0.0217^{**}$ (0.01)	$0.0262^{***}$ (0.01)	$0.0270^{***}$ (0.01)	$0.0344^{***}$ (0.01)		$0.0275^{**}$ (0.01)	$0.0607^{***}$ (0.02)	$0.0679^{***}$ (0.02)	$0.0450^{**}$ (0.02)	$0.108^{***}$ (0.04)
$\varepsilon * \overline{mrpk}$						$0.080^{**}$ (0.04)					
Observations	3692188	3538432	2641657	2641657	2641657	2641657	3290824	401359	40932	1253505	1970415
$R^2$	0.020	0.255	0.295	0.295	1.000	1.000	0.020	0.035	0.063	0.033	0.026
Time-sector FE	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
Firm FE	NO	YES	YES	YES	YES	YES	NO	NO	NO	NO	NO
Firm Controls	NO	NO	YES	YES	YES	YES	NO	NO	NO	NO	NO
Agg. Control	NO	NO	NO	YES	YES	YES	NO	NO	NO	NO	NO
MP shock*Firm Control	NO	NO	NO	NO	YES	YES	NO	NO	NO	NO	NO
MRPK demeaning	NO	NO	NO	NO	NO	YES	NO	NO	NO	NO	NO
Panel	FULL	FULL	FULL	FULL	FULL	FULL	EMP < p90	EMP > p90	LARGE	N > 5	FULL

Table 4: Robustness

Notes: This table reports the results of estimating equation (43), departing from some of the specifications of the estimation in the main text of equation (41). Column (1) is the baseline regression, that is, that of equation (41). Column (2) includes firm fixed-effects, and Column (3) also includes lagged firm-level controls (total assets, sales growth, leverage, capital growth and net short term financial assets). Column (4) nus the same specification as Column (3), but adding the interaction of log  $(MRPK_{j,t-1})$  and lagged GDP growth. Column (5) adds to the specification of column (4) the interaction of lagged firm level controls and the mometary policy shock. Column (6) runs the same specification as Column (5), just replacing the main variable of interest log  $(MRPK_{j,t-1})$  with its demeaned value at the firm level. Columns (7), (8) and (9) run the baseline specification, but including only firms with less employees than 90th percentile (15 employees), firms with more employees than 90th percentile, and firms with more than 100 employees, respectively. Column (10) shows the baseline specification, but only for firms that we observe at least for 6 consecutive years (from t-1 to t+4). Column (11) runs the baseline specification from (1), but using the log change of profits as dependent variable. Standard errors are clustered at the sector-year level.