Firm Heterogeneity, Capital Misallocation and Optimal Monetary Policy

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Capital misallocation and real interest rates

- Reis (2013), Gopinath et al. (2017), Asriyan et al. (2021) $\Rightarrow$ a decline in real interest rates worsens capital misallocation:

$$\downarrow r \Rightarrow \downarrow TFP$$

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   - New Keynesian literature (Evans (1992), Christiano et al. (2005), Garga and Singh (2021), Jordà et al. (2020), Moran and Queralto (2018), Meier and Reinelt (2020), Baqaee et al. (2021)...) $\Rightarrow$ expansionary monetary policy improves TFP:

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2. How does it affect the design of optimal monetary policy?

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- Expansionary monetary policy increases share of investment by high-MRPK firms $\Rightarrow$ increases TFP...
  - Supported by empirical analysis based on Spanish firm-level micro data

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- New algorithm to solve nonlinearly for Ramsey optimal policies with heterogeneous agents in continuous time.

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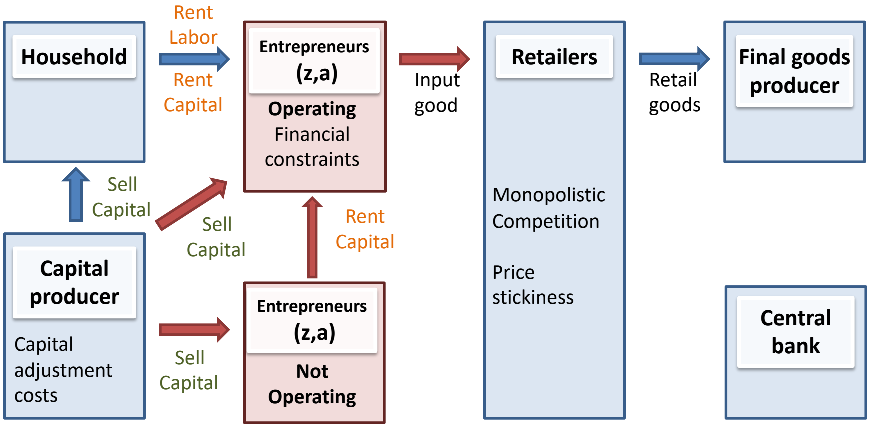
- New algorithm to solve nonlinearly for Ramsey optimal policies with heterogeneous agents in continuous time.
- Optimal monetary policy:
  - Misallocation creates a *time inconsistent* motive to temporarily expand the economy.
  - Timeless response to efficient shocks: “divine coincidence” holds ...
  - ... but at the ZLB: low for even longer.

Model

The model in a nutshell



Heterogeneous entrepreneurs

- Heterogeneity in entrepreneurs' net worth (a_t) and productivity (z_t; follows OU-diffusion process).
- Firms produce the input good with a CRS technology using labor (l_t) and capital (k_t).
- Entrepreneurs can borrow capital $b_t = k_t - a_t$, subject to a borrowing constraint $k_t \leq \gamma a_t$.
- Entrepreneurs can pay dividends d_t or accumulate net worth a_t; they retire at rate η.
- Entrepreneurs are household's members (as in Gertler & Karadi, 2011, unlike Moll, 2014).

Entrepreneur's problem

$$V_0(z,a) = \max_{\dot{a}_t, d_t \geq 0} \mathbb{E}_0 \int_0^\infty e^{-\int_0^t (r_s+\eta) ds} \left(d_t + \overbrace{\eta q_t a_t}^{\text{liquidation value}} \right) dt$$

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s.t.

$$\dot{a}_t q_t + d_t = \underbrace{\left(\overbrace{\max\{\Phi_t(z), 0\}\gamma}^{\text{operating profits}} + \overbrace{\left(\frac{R_t - \delta q_t}{q_t} \right)}^{\text{return on capital}} \right)}_{S_t(z)} q_t a_t$$

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$$\begin{aligned} \Phi_t(z_t, a_t) &= \max_{k_t, l_t} \{ m_t (z_t k_t)^\alpha (l_t)^{1-\alpha} - w_t l_t - R_t k_t \} \\ s.t.\ k_t &\leq \gamma a_t \end{aligned}$$

- d_t: dividends
- R_t: rental rate of capital
- δ: capital depreciation
- q_t: price of capital
- a_t: net worth (capital owned by firm)
- r_t: real interest rate

Solution

$$k_t(z,a) = \begin{cases} \gamma a, & \text{if } z \geq z_t^*, \\ 0, & \text{if } z < z_t^*, \end{cases}$$

$$z_t^* = \frac{R_t}{\alpha \left(\frac{(1-\alpha)}{w_t}\right)^{(1-\alpha)/\alpha} m_t^{\frac{1}{\alpha}}} = \frac{R_t}{\varphi_t}$$

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- If $z < z_t^*$, optimal size is $k_t(z,a) = k_t^*(z) = 0 \rightarrow$ Entrepreneur is unconstrained
  - She lends her net worth to other entrepreneurs.

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- Entrepreneurs optimally never distribute dividends until liquidation.
  - Intuition: return of funds inside the firm is always at least the real rate $\left(\frac{R_t - \delta q_t}{q_t}\right)$, and the liquidation value of the firm is all its net worth .

Distribution in net worth shares

- The evolution of the joint distribution of net worth and productivity $g_t(z,a)$ is given by the KFE:

$$\frac{\partial g_t(z,a)}{\partial t} = \underbrace{-\frac{\partial}{\partial a}[g_t(z,a)s_t(z)a]}_{\text{Entrepreneurs' savings}} \underbrace{-\frac{\partial}{\partial z}[g_t(z,a)\mu(z)] + \frac{1}{2}\frac{\partial^2}{\partial z^2}[g_t(z,a)\sigma^2(z))]}_{\text{idiosyncratic TFP shocks}}$$

$$\underbrace{-\eta g_t(z,a)}_{\text{Entrepreneurs retire}} \quad \underbrace{+\eta g_t(z,a/\psi)/\psi}_{\text{New entrepreneurs}}$$

- Only need the distribution of net worth shares $\omega_t(z) = \frac{1}{A_t}\int_0^\infty a g_t(z,a)da$.

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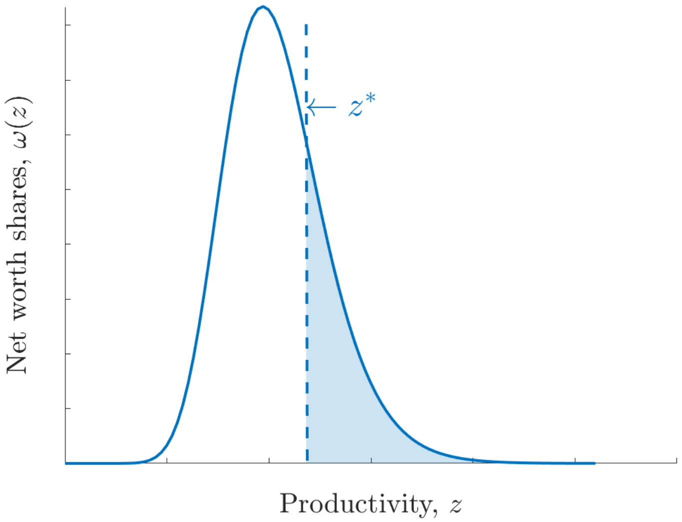
- Model is isomorphic to standard RANK with endogenous TFP $\tilde{Z}_t$.
- Aggregate output Y_t is

$$Y_t = \tilde{Z}_t K_t^\alpha {L_t}^{1-\alpha}, \quad \tilde{Z}_t = \left(\mathbb{E}_{\omega_t(z)}\left[z \mid z > z_t^*\right]\right)^\alpha.$$

Aggregation



The Dynamics of Misallocation

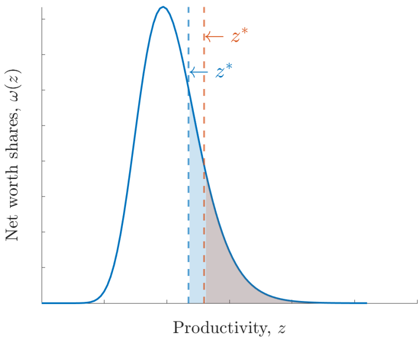
Channels affecting misallocation

$$\tilde{Z}_t = \left(\mathbb{E}_{\omega_t(z)}\left[z \mid z > z_t^*\right]\right)^{\alpha}$$

Productivity-threshold channel

Changes in the share of constrained firms

$$z_t^* \varphi_t = R_t \text{ where } \varphi_t = \alpha \left(\frac{(1-\alpha)}{w_t}\right)^{\frac{1-\alpha}{\alpha}} m_t^{\frac{1}{\alpha}} \text{ (avg. MRPK)}$$



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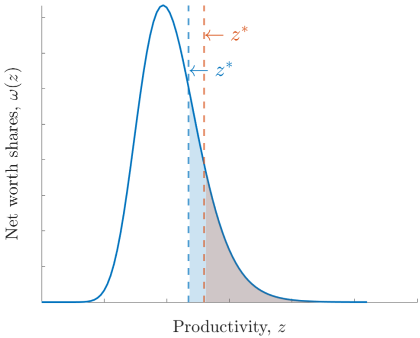
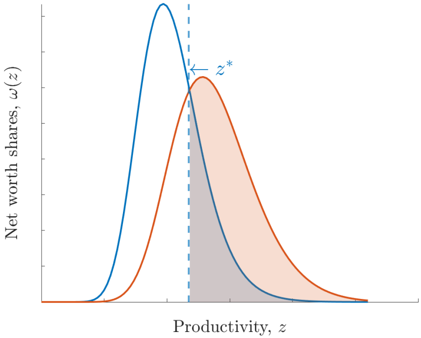


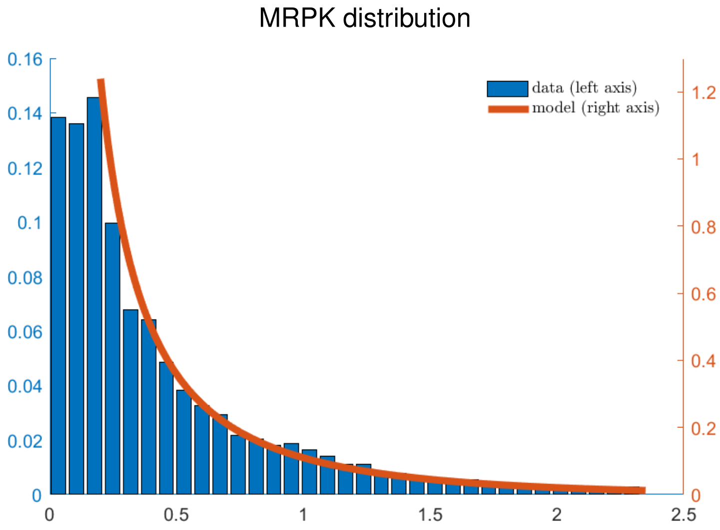
Net-worth distribution channel

Changes in the distribution of net-worth across firms

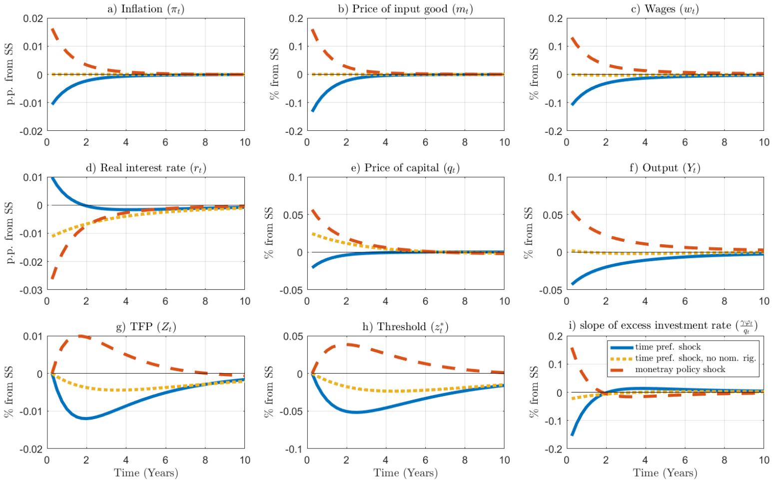
$$\tilde{\Phi}_t(z) = \frac{\gamma}{q_t}\left(z_t \varphi_t - R_t\right)$$



Calibration



Natural rates, real rates, and misallocation



Optimal Monetary Policy

Central Bank's Ramsey problem

$$\max_{\{\omega_t(z),\, \text{Prices}_t, \text{Quantities}_t\}_{t\in\,[0,\,\infty)}} \mathbb{E}_0 \int_0^{\infty} e^{-\rho^h t} u(C_t, L_t) dt$$

subject to private equilibrium conditions $\forall t \in [0, \infty)$ and initial conditions

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- Need to keep track of the whole distribution of firms $\omega_t(z)$

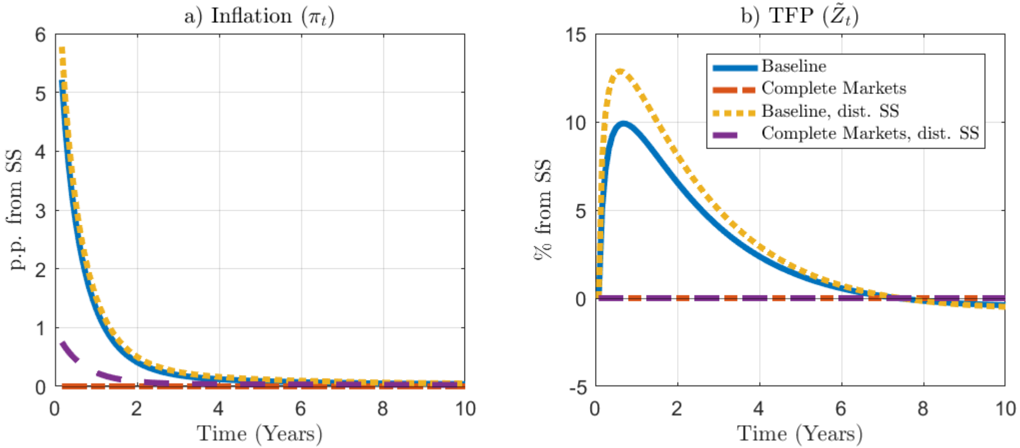
Sketch of solution algorithm

1. Discretize the time space (Δt); and the state space (Δz) into J grid points using finite differences (Achdou et al, 2017):
   - system of 2J equations and 2J unknowns for the HJB and the KFE equation (we don't have a HJB).

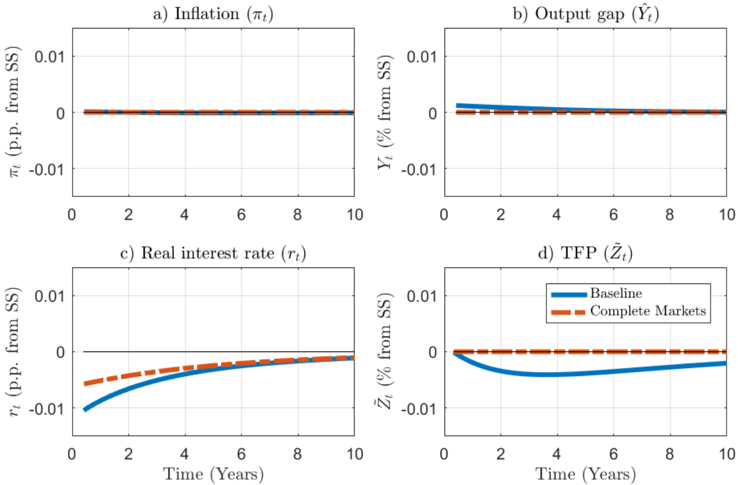
$$\begin{pmatrix} \frac{1}{\Delta t}\left(\mathbf{v}^{n+1} - \mathbf{v}^{n}\right) + \rho \mathbf{v}^{n+1} & = \mathbf{u}^{n+1} + \mathbf{A}^{n+1}\mathbf{v}^{n+1} \\ \frac{\mathbf{g}^{n+1} - \mathbf{g}^{n}}{\Delta t} & = \left(\mathbf{A}^{n+1}\right)^{T} \mathbf{g}^{n+1} \end{pmatrix}$$

   - set of X equilibrium conditions (MC, FOCs of representative agents)
2. Compute the planner's optimality conditions on discretized problem : $(2J + X) + (2J + X + 1)$ equations using symbolic differentiation
3. Solve the transitional dynamics up to horizon T using a Newton algorithm to solve a large equation set of $[(2J + X) + (2J + X + 1)]\ T$ equations (cf. Auclert et al., 2020)

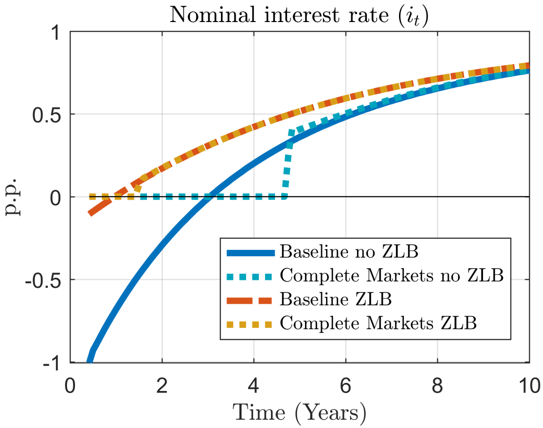
Optimal Ramsey policy: a new time inconsistency



Timeless optimal response: (quasi-)divine coincidence holds



Timeless optimal response with ZLB: low for even longer



Empirical Evidence

1. After a monetary policy expansion, high-MRPK firms increase their investment relatively more

- Data: yearly balance sheet and cash flow data for the quasi-universe of Spanish firms
- Local projection

$$\log k_{j,t} - \log k_{j,t-1} = \beta_0 + \beta_1 \log \left(MRPK_{j,t-1} \right) + \beta_2 \log \left(MRPK_{j,t-1} \right) \varepsilon_t^{MP} + \gamma_{s,t} + u_{j,t},$$

- ε_{t-1}^{MP} monetary policy shocks from Jarociński and Karadi (2020)

Table: Response of firm-level investment to an expansionary monetary policy shock

	(1)	(2)
β_2	0.028^{***}	0.044^{***}
	(0.01)	(0.02)
Obs	$3,692,188$	$1,253,505$
R^2	0.02	0.03
$\gamma_{i,t}$	Yes	Yes
Panel	Full	$N_i > 5$

Notes: The first column displays the OLS estimate of β_2 for the full unbalanced panel. The second column reports the same specification, but restricting the sample to firms that we observe for more than 5 consecutive years. Both regressions include sector-year fixed effects and standard errors clustered at the sector-year level.

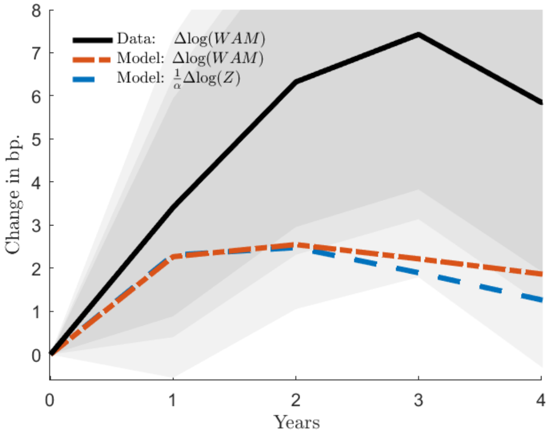
2. After a monetary policy expansion, misallocation is reduced

- Isolate contribution of capital reallocation to TFP by computing the dynamics of weighted average MRPK with constant initial firm-level MRPK

$$WAM_{t,\tau} = \sum_{j=0}^{J} MRPK_t^j \frac{k_{t+\tau}^j}{K_{t+\tau}}$$

- Local projection (at sector level)

$$\Delta \log WAM_{t,\tau,s} = \alpha_{s,\tau} + \beta_\tau \varepsilon_t^{MP} + u_{j,t,\tau}$$



Data: $\Delta\log(WAM)$
Model: $\Delta\log(WAM)$
Model: $\frac{1}{\alpha}\Delta\log(Z)$
Change in bp.
Years
0
1
2
3
4
5
6
7
8

Appendix

Representative household

Back

Standard consumption-labor-savings choice

$$\max_{C_t, L_t, D_t, B_t^N} \mathbb{E}_0 \int_0^\infty e^{-\rho^h t} u(C_t, L_t) dt$$

$$s.t.$$

$$\dot{D}_t q_t + \dot{B}_t^N + C_t = (R_t - \delta q_t) D_t + (i_t - \pi_t) B_t^N + w_t L_t + T_t$$

- C_t: consumption
- D_t: capital holdings
- B_t^N holdings of nominal bonds (zero net supply)
- L_t: labor supply
- i_t: nominal interest rate
- T_t : profits of *retailers*, *capital good producer* and *net dividends* from firms

Capital good producer

Produces capital and sells it to the household and the firms at price q_t

Back

$$\max_{\iota_t, K_t} \mathbb{E}_0 \int_0^\infty e^{-\int_0^t r_s ds} \left(q_t \iota_t - \iota_t - \Xi\left(\iota_t\right)\right) K_t dt.$$

$$s.t. \quad \underbrace{\dot{K}_t = \left(\iota_t - \delta\right) K_t}_{\text{LOM of } K_t}.$$

- ι_t: investment *rate*,
- $\Xi\left(\iota_t\right) = \frac{\phi^k}{2}\left(\iota_t - \delta\right)^2$: quadratic adjustment costs.

New Keynesian block

Back

- Final good producers aggregate varieties $j \in [0,1]$. Cost minimization implies demand for variety j is given by

$$y_{j,t}\left(p_{j,t}\right) = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t, \text{ where } \underbrace{P_t = \left(\int_0^1 {p_{j,t}}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}}_{\text{Agg. Price index}}.$$

- Retailers maximize

$$\max_{p_{j,t}} \int_0^\infty e^{-\int_0^t r_s ds} \left\{ \underbrace{\left(\frac{p_{j,t}}{P_t} - m_t\right)}_{\text{Mark-up}} \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t - \frac{\theta}{2}\left(\frac{\dot{p}_{j,t}}{p_{j,t}}\right)^2 Y_t \right\} dt$$

  - ε: elasticity of substitution across goods $\varepsilon > 0$.
  - $p_{j,t}$: price of variety j.
  - θ: price adjustment cost parameter.

New Keynesian block

Back

- The symmetric solution to the pricing problem yields the New Keynesian Phillips curve

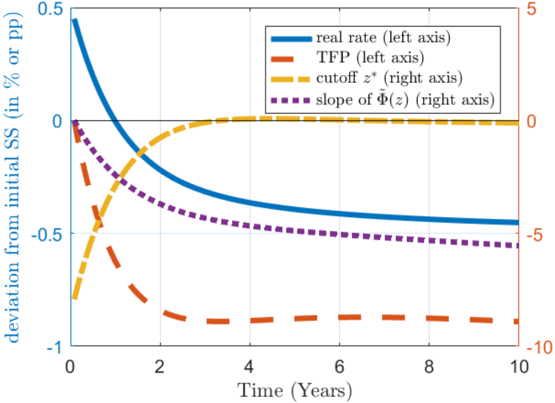
$$\left(r_t - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\varepsilon}{\theta}\left(m_t - m^*\right) + \dot{\pi}_t, \qquad m^* = \frac{\varepsilon - 1}{\varepsilon},$$

- $\pi_t = \frac{\dot{P}}{P_t}$ is inflation,
- m_t are relative prices of intermediate good (inverse mark-ups of retailers),
- m^* is the optimal inverse mark-up,
- Real rates are defined as $r_t \equiv \frac{R_t - \delta q_t + \dot{q}_t}{q_t}$.

Back

$$
\begin{aligned}
w_t =& (1-\alpha) m_t Z_t K_t^{\alpha} L_t{}^{-\alpha}, \\
R_t =& \alpha m_t Z_t K_t^{\alpha-1} L_t{}^{1-\alpha} \frac{z_t^*}{\mathbb{E}\left[z \mid z > z_t^*\right]}, \\
\frac{\dot{A}_t}{A_t} =& \frac{1}{q_t}\left[\gamma(1-\Omega(z_t^*))\left(\alpha m_t Z_t K_t^{\alpha-1} L_t{}^{1-\alpha} - R_t\right) + R_t - \delta q_t - q_t(1-\psi)\eta)\right].
\end{aligned}
$$

Permanent decrease in natural rates *decreases* TFP



RANK vs HANK

Back

RANK

- All capital is owned by HH $D_t = K_t$
- No financial frictions.
- TFP is exogenous
  $Z = 1$

HANK

- Capital is owned by HH and entrepreneurs: $D_t + A_t = K_t$
- Financial frictions: $k_t \leq \gamma a_t$
- TFP is endogenous
  $Z = (\mathbb{E}_t [z \mid z > z_t^*])^{\alpha}$

- Introduce subsidies in both economies, such that the SS mark-up distortion is undone.