J Econ Growth
DOI 10.1007/s10887-011-9063-4

Optimal research and development and the cost of business cycles

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Abstract While research and development (R&D) investment has been procyclical in the post-war period, recent literature suggests that the optimal path for R&D is countercyclical, and that the economy would be better off by subsidizing R&D in recessions. The objective of this paper is to analyze the welfare effects of distortions in the intertemporal allocation of R&D resources and to compare diverse policy interventions so as to improve social welfare. To this end, we introduce a calibrated dynamic stochastic general equilibrium model with Schumpeterian endogenous growth that is capable of explaining the observed procyclicality of R&D. Our results show that the cost of business cycles is lower in the decentralized economy with procyclical R&D than in the efficient allocation with countercyclical R&D. This is because the suboptimal propagation of shocks in the decentralized equilibrium offsets some of the existing steady-state distortions. In this *second-best* context, countercyclical R&D subsidies have no positive effect on welfare. In contrast, fiscal policies aimed at restoring the optimal steady-state produce large welfare gains.

Keywords Schumpeterian growth · Technology adoption · Optimal subsidy

JEL classification E32 · O38 · O40

1 Introduction

The literature on endogenous growth has underlined how several market failures in a decentralized economy may produce an inefficient equilibrium allocation of research and development (R&D) resources, as entrepreneurs do not internalize all the consequences of their R&D investments.[1] In principle, this misallocation may affect not only the steady-state level of

[1] See, for example, Romer (1990), Grossman and Helpman (1991) or Aghion and Howitt (1992). According to Jones and Williams (1998, 2000), a decentralized economy typically underinvests in R&D relative to what is socially optimal.

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Published online: 26 April 2011

R&D investment, but also its intertemporal dynamics. In this case, the propagation of aggregate shocks in the decentralized economy may differ from the first-best allocation, which may potentially affect the welfare impact of business cycles. The objective of this paper is to explore the welfare effects of the intertemporal allocation of R&D resources in a model consistent with the observed cyclical behavior of R&D.

R&D has been procyclical over the post-war period in the United States (US). This evidence contrasts with the traditional "Schumpeterian" notion that recessions should promote activities that contribute to long-run growth. The procyclicality of R&D has been discussed in a number of papers. For example, Comin and Gertler (2006) explore how procyclical R&D can explain the low-frequency fluctuations observed in many industrialized economies and Fatas (2000) discusses how procyclical R&D affects the link between long-run growth rates and the persistence of business cycles. The paper that most closely relates to ours is Barlevy (2007), who finds that the procyclicality of R&D may be the consequence of dynamic externalities in R&D that make entrepreneurs concentrate their innovation in booms even if it is socially optimal to concentrate it in recessions. In his model, R&D is procyclical in the decentralized economy *but* countercyclical in the efficient allocation. Barlevy suggests that procyclical R&D may amplify the persistence of aggregate shocks and increase the cost of business cycles in the decentralized economy.[2]

This paper relates to the above literature by analyzing two main issues. First, we compute welfare measures under different allocations of R&D expenditures, decomposing welfare effects into steady-state and dynamic components. In particular, we compare the welfare cost of business cycles in the decentralized equilibrium and in the counterfactual efficient allocation. While papers including Lucas (1987) and Barlevy (2004) have calculated the welfare costs of business cycles in economies where the steady-state is Pareto-optimal, here the steady-state in the decentralized economy is suboptimal and, hence, the cost of business cycles in the decentralized economy is different from the cost in the first-best allocation. Indeed, we focus on this difference to analyze whether the procyclicality of R&D introduces additional costs, as suggested by Barlevy (2007), or not.

Second, we analyze what type of policies are more effective from a welfare point of view in a context of market failures that affect both the steady-state *and* the dynamic equilibrium. Howitt and Aghion (1998) propose fiscal policies that restore the optimal steady-state. In contrast, Barlevy (2007) suggests that the economy would be better off reallocating R&D resources from expansions to recessions to thus reduce the procyclicality of R&D. This can be done, for example, with a countercyclical R&D subsidy. This paper compares the welfare gains of these two policies.

To analyze these issues, we develop a dynamic stochastic general equilibrium (DSGE) model that incorporates Schumpeterian endogenous growth and knowledge spillovers *à la* Howitt and Aghion (1998). We calibrate a version of the model that reasonably replicates the dynamic and long-run features of the post-war US economy and use it to perform the welfare analysis and counterfactual experiments. The model includes vertical innovations and business-stealing effects as in Barlevy (2007). In addition, the model incorporates endogenous labor supply, which might contribute to explaining the procyclicality of R&D as suggested by Fatas (2000), and it employs a "lab-equipment" specification where R&D activities require both labor and capital inputs in the form of final goods, so that the equilibrium R&D is procyclical. Finally, the model provides a framework to analyze the impact of diverse assumptions about long-run productivity growth.

[2] Other relevant papers that analyze the procyclicality of R&D are Aghion and Saint-Paul (1998), Aghion et al. (2005) and Francois and Lloyd-Ellis (2009).

Our results show that the costs of business cycles are *higher* in the Pareto-optimal allocation with countercyclical R&D than in the decentralized economy with procyclical R&D. This apparently counterintuitive result is due to the fact that, in this setup, the distorted propagation of shocks in the decentralized equilibrium partially mitigates some of the welfare losses of the suboptimal steady-state. The cost of business cycles in the case of a decentralized economy (8.5%) is in the range found by Barlevy (2004) (8–10%), and much higher than the cost found by Lucas (1987).[3]

With regard to welfare-improving policies, we find that a time-varying subsidy that replicates the countercyclicality of optimal R&D has no positive effects on welfare. This is a consequence of the *theory of the second-best* of Lipsey and Lancaster (1956), which states that a policy that only addresses one of the multiple distortions in the economy is not necessarily welfare-improving. We also demonstrate that the Pareto-optimal steady-state can always be restored by introducing two subsidies, one to capital accumulation and the other to R&D, both financed by lump-sum taxes on households. The decentralized economy with a restored steady-state is also inefficient, as evidenced by R&D remaining more procyclical than in the first-best allocation. Despite this inefficiency, the welfare in this case is similar to that in the efficient allocation. All these results call for caution when considering the conclusion of Barlevy (2007) about the optimality of fiscal policies that modify the time-path of R&D.

We pay special attention to the robustness of the results under different assumptions about long-run technology growth. Our benchmark model assumes, following Howitt and Aghion (1998), that the technology frontier evolves endogenously due to knowledge spillover effects of domestic R&D. However, there are reasons to believe that the technology frontier should be taken as exogenous at the level of an individual country.[4] Depending on the magnitude of these knowledge spillovers, the optimal steady-state level of R&D investment could be above, or below, the level in the decentralized economy, as discussed by Aghion and Howitt (1992). Nevertheless, we show how, irrespective of the magnitude of the spillovers, the cost of business cycles is always lower in the decentralized equilibrium than in the efficient allocation.

Beyond the results already mentioned, this paper makes two additional contributions to the literature on the technical front. First, it introduces a new algorithm to compute social welfare, based on Monte Carlo simulations, especially suitable for a model in which variables are nonstationary. Second, it provides a model that analyzes growth at both business-cycle and lower frequencies, which is necessary to consistently take DSGE models to the data, as discussed by Canova and Ferroni (2009).

The structure of the paper is as follows. In Sect. 2 we introduce the decentralized model and the counterfactual efficient allocation. In Sect. 3 we derive results regarding the long-run behavior of the economy. In Sect. 4 we analyze the dynamic properties of the model. In Sect. 5, we explore how our results change depending on the assumptions about the magnitude of knowledge spillovers. Finally, we present our conclusions in Sect. 6.

2 The model

The model integrates endogenous growth into an otherwise conventional real business-cycle (RBC) model. Endogenous growth is based on vertical innovations as in Howitt and Aghion (1998). Producers of final goods use labor and a continuum of intermediate goods as inputs.

[3] We find these large differences even though shocks in our model only affect the level of the growth trend of consumption, whereas in Barlevy (2004) shocks affect the growth rate.

[4] See for example, Porter and Stern (2000), Howitt (2000) or Comin (2004).

These intermediate goods differ in their productivity and each of them is produced by a monopolistic competitive firm using capital. The amount of capital necessary to produce each intermediate good is proportional to its productivity, thus reflecting the fact that more advanced products require increasingly capital-intensive techniques. In each period, there is a probability that the productivity of an intermediate good jumps to the technology frontier owing to the R&D activities of entrepreneurs. Entrepreneurs borrow resources and invest them in R&D in an attempt to increase their probabilities of making a discovery. If a discovery occurs, the successful entrepreneur introduces a new enhanced intermediate product in her sector and becomes the new monopolist until she is replaced by another entrepreneur. The technology frontier (i.e., the productivity level of the most advanced sector) evolves endogenously as the result of positive spillovers from innovation activities.

We choose this model, in the spirit of Kydland and Prescott (1982), because it provides a good explanation of long-term growth and cross-country income differences, as discussed by Aghion and Howitt (1998) and Howitt (2000), and it thus seems an appropriate tool to study aggregate fluctuations.[5] It is a model with vertical innovations and business-stealing effects, in contrast to Comin and Gertler (2006) where endogenous growth is due to an expanding variety of goods *à la* Romer (1990). In addition, the model assumes "lab-equipment" R& D. Aghion and Howitt (1998, pp. 93–102) justify lab-equipment R&D by highlighting how research uses a great deal of physical capital in the form of laboratories, computers and specific machinery. Howitt and Aghion (1998) provide more specific empirical evidence of the important role of capital in R& D. Comin and Gertler (2006) also consider a "lab-equipment" specification of the R&D process. In this case, the opportunity costs of R&D are constant over the cycle, as they depend on the price of final goods, whereas the benefits of an innovation are proportional to the profits and hence they are higher during expansions, as noted by Aghion and Saint-Paul (1998). The literature has presented alternative mechanisms to generate procyclical R&D: Barlevy (2007) considers a fixed cost of production in the intermediate goods sector, Aghion et al. (2005) introduce credit frictions, Francois and Lloyd-Ellis (2009) focus on endogenous implementation cycles and Fatas (2000) emphasizes the procyclicality of labor supply.

In this section, we introduce the model, characterize its equilibrium conditions, and present the "benevolent" social planner solution. Throughout the paper we denote with capital letters (e.g., S_t) the nonstationary variables whereas we reserve the lowercase letters (e.g., s_t) for stationary variables. Variables in steady-state are denoted without the time subscript (e.g., s).

2.1 Final goods

Final goods are produced under perfect competition using labor and a continuum of intermediate products. Final-goods firms maximize their profits

$$\max_{l_t, m_{j,t}} Y_t - W_t l_t - \int_0^1 P_{j,t} m_{j,t} dj$$

[5] Additionally, Howitt and Mayer-Foulkes (2005) employ a small variation of the model to explain the divergence in per-capita income that has taken place among countries since the mid-nineteenth century and the convergence that took place among the richest countries during the second half of the twentieth century.

subject to

$$Y_t = l_t^{1-\alpha}\left(\int_0^1 A_{j,t} m_{j,t}^{\alpha} dj\right), \quad (1)$$

where $m_{j,t}$ is the amount of intermediate product $j \in [0, 1]$, l_t is labor supply, and $A_{j,t}$ is a productivity parameter attached to the latest version of intermediate product j. The price of final output is normalized to 1. The first-order conditions are given by

$$P_{j,t} = \alpha A_{j,t} l_t^{1-\alpha} m_{j,t}^{\alpha-1}, \quad (2)$$

$$W_t = (1-\alpha)\frac{Y_t}{l_t}. \quad (3)$$

2.2 Intermediate goods

Final output can be used interchangeably as a consumption, a capital good, or an input to innovation. Each intermediate product is produced by an incumbent monopolist using capital, according to the production function:

$$m_{j,t} = K_{j,t-1}/A_{j,t}, \quad (4)$$

where $K_{j,t-1}$ is the capital in sector j at time t, installed in period $t-1$. Division by $A_{j,t}$ indicates that successive vintages of the intermediate product are produced by increasingly capital-intensive techniques. The incumbent monopolist of each sector solves the problem

$$\max_{m_{j,t}} \left(P_{j,t} m_{j,t} - q_t K_{j,t-1}\right) \quad (5)$$

subject to (2) and (4), where q_t is the rental cost of capital. Marginal costs and marginal revenues are proportional to $A_{j,t}$. Therefore all intermediate producers choose to supply the same amount of intermediate product m_t.[6] The aggregate capital in the economy is $K_{t-1} = \int_0^1 K_{j,t-1} dj = m_t A_t$, where $A_t = \int_0^1 A_{j,t} dj$ is the average productivity across all sectors in final-goods production. As a result, the aggregate production function of the economy (1) can be reduced to $Y_t = K_{t-1}^{\alpha}(A_t l_t)^{1-\alpha}$.

The cost of capital q_t can be expressed as a function of the aggregate level of capital

$$q_t = \alpha^2 \frac{Y_t}{K_{t-1}}, \quad (6)$$

and the flow of profits that each incumbent earns is

$$\Pi_t(A_{j,t}) = \frac{\alpha(1-\alpha)Y_t}{A_t} A_{j,t}, \quad (7)$$

so that a share $(1-\alpha)$ of final output is allocated to wages, α^2 to capital costs and $\alpha(1-\alpha)$ to profits.

2.3 Productivity

Innovations result from entrepreneurship using technological knowledge. At any date there is a "technology frontier" that represents the most advanced technology across all sectors:

$$A_t^{\max} \equiv \max\{A_{j,t} \mid j \in [0, 1]\}. \quad (8)$$

6 $m_t = \left(\frac{\alpha^2}{q_t}\right)^{1/(1-\alpha)} l_t$.

In each period, there is a probability $n_{j,t}$ that the productivity $A_{j,t}$ of an intermediate good in sector j jumps to the technology frontier due to the R&D activities of an entrepreneur in this sector. Entrepreneurs invest resources in R&D activities in an attempt to increase their probabilities of making a discovery and replace the current incumbents. If a discovery occurs, the successful entrepreneur introduces a new enhanced intermediate product in her sector and becomes the new monopolist until she is replaced by another entrepreneur.

Therefore, in each period, the productivity in sector j evolves according to

$$A_{j,t+1} = \left\{ \begin{array}{ll} A_t^{\max}, & \text{with probability } n_{j,t} \\ A_{j,t}, & \text{with probability } 1 - n_{j,t} \end{array} \right\}. \tag{9}$$

Once an innovation occurs, it results in an improved version of the existing product by raising its productivity $A_{j,t+1}$ to the technology frontier $A_t^{\max}$. The entrepreneur then enters into Bertrand competition with the previous incumbent in that sector, who by definition produces a good of inferior quality. Rather than facing a price war with a superior rival, the incumbent exits. Having exited, the former incumbent cannot threaten to re-enter. Therefore, in period $t+1$ the former entrepreneur becomes the new incumbent.[7] These dynamics explain why incumbents can always charge the unconstrained monopolistic price without worrying about competition from earlier vintages of the product.[8]

Entrepreneurs must incur a fixed R&D cost $X_{j,t}$, measured in units of final output of price 1. The probability $n_{j,t}$ is a function of this quantity $X_{j,t}$ of output devoted to R&D in the sector:

$$n_{j,t} = \left\{ \begin{array}{ll} \left(\frac{X_{j,t}}{\lambda A_t^{\max}}\right)^{\frac{1}{(\eta+1)}} & \text{if } X_{j,t} < \lambda A_t^{\max} \\ 1 & \text{if } X_{j,t} \geq \lambda A_t^{\max} \end{array} \right\}, \quad \eta \geq 0. \tag{10}$$

Equation 10 shows decreasing returns to scale in innovation.[9] Parameter λ accounts for the productivity of resources devoted to R&D and η determines the extent of decreasing returns.[10] The amount of resources is adjusted by the technology frontier variable $A_t^{\max}$ to represent the increasing complexity of progress: as technology advances, the resource cost of further advances increases proportionally.

2.4 Entrepreneurs

The value of being the incumbent in period t in a sector with productivity $\bar{A}$, $V_{j,t}(\bar{A})$ is the discounted flow of profits that the incumbent may obtain by taking into account the probability of obsolescence due to the arrival of a new innovation in this sector; therefore

$$V_{j,t}(\bar{A}) = \Pi_t(\bar{A}) + \frac{(1 - n_{j,t})}{r_t} E_t \left[V_{j,t+1}(\bar{A}) \right], \tag{11}$$

[7] We allow for an implementation lag of one period in order to capture all these dynamics. Longer implementation lags may also be plausible, but we leave this analysis for further research.

[8] This is the usual assumption. See Aghion and Howitt (1998, Chap. 12) or Howitt and Aghion (1998) for further explanations.

[9] Previous studies have found decreasing returns in R&D expenditure; e.g. Kortum (1993). Aghion and Howitt (1998, Chap. 12) also consider decreasing returns to R&D. This is an extension of the Howitt and Aghion (1998) model, which only considers constant returns to R&D.

[10] For the diverse calibrations considered in this paper, n_t is always less than 1.

where r_t is the inverse of the price of a risk-free bond.[11] The first term reflects the flow of profits of the monopolist whereas the second term is the discounted value of still being the incumbent at $t+1$.

We consider that, in each period, there is a single entrepreneur in each sector. Her problem is to maximize the discounted value of becoming the incumbent in the next period $E_t\left[V_{j,t+1}(A_t^{\max})\right]/r_t$, weighted by the probability of doing so $n_{j,t}$ (since the success of her R&D activities is uncertain), namely

$$\max_{X_{j,t}} \frac{n_{j,t}}{r_t} E_t\left[V_{j,t+1}(A_t^{\max})\right] - X_{j,t}, \tag{12}$$

subject to (10). The entrepreneur incurs a fixed cost $X_{j,t}$ that affects the success probability according to (10). All entrepreneurs face the same problem, since the value of becoming the incumbent in the next period $V_{j,t+1}(A_t^{\max})$ is the same for all sectors; therefore, the input invested in R&D in each intermediate sector is the same: $X_{j,t} = X_t$ and $n_{j,t} = n_t$.

The first-order condition is that the marginal costs of an extra unit of goods allocated to research equals the discounted marginal expected benefit and hence

$$X_t = \frac{n_t E_t\left[V_{t+1}(A_t^{\max})\right]}{(\eta+1)r_t}. \tag{13}$$

In the case $\eta = 0$ (linear R&D technology), this equation is equivalent to the free-entry condition so that the total amount of resources invested in R&D equals the expected value of becoming the next monopolist.

The growth of the technology frontier $A_t^{\max}$ occurs as a result of the knowledge spillovers produced by innovations, as in Aghion and Howitt (1992). At any moment in time, the technology frontier is available to any successful innovator, and this publicly available knowledge grows at a rate proportional to the aggregate rate of innovations. Each innovation endogenously "pushes" the technology frontier by a factor $(1+\sigma_t)$. Therefore

$$g_t \equiv \frac{A_t^{\max}}{A_{t-1}^{\max}} = \int_0^1 \left[n_{j,t-1}(1+\sigma_t) + (1-n_{j,t-1})1\right] dj = 1 + \sigma_t n_{t-1}, \tag{14}$$

where σ_t is the knowledge spillover coefficient. We assume it follows a process $\log(\sigma_t) = \log(\sigma) + \varepsilon_t^\sigma$ with $\varepsilon_t^\sigma \overset{i.i.d.}{\sim} N(0, \sigma_\sigma)$. We consider an alternative specification in Sect. 5.

2.5 Households

The representative household solves

$$\max_{C_t, I_t, B_t, K_t, l_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \mu_t \frac{l_t^{1+\psi} - 1}{1+\psi}\right], \tag{15}$$

with $0 < \beta < 1$, subject to

$$C_t + I_t + \frac{B_t}{r_t} = W_t l_t + q_t K_{t-1} + D_t + B_{t-1}, \tag{16}$$

$$K_t = I_t + (1-\delta)K_{t-1}, \tag{17}$$

[11] Therefore, the risk-free interest rate is $\log(r_t) \approx r_t - 1$. Up to a first-order approximation, discounting by the risk-free interest rate is equivalent to using the stochastic discount factor. The results in this paper do not depend on whether we employ one discounting method or the other.

where C_t is consumption, I_t is investment, B_t is the amount of state-contingent bonds and D_t are the dividends from a collective investment fund. This investment fund finances entrepreneurs' R&D investments in exchange for the ownership of the new incumbent firms from which it collects the profits $D_t = \int_0^1 \left(\Pi_t(A_{j,t}) - X_{j,t}\right) dj = \alpha(1-\alpha)Y_t - X_t$. We consider a labor shock so that μ_t is a stochastic process $\log(\mu_t) = (1-\rho_\mu)\log(\mu) + \rho_\mu \log(\mu_{t-1}) + \varepsilon_t^\mu$ with $\varepsilon_t^\mu \overset{i.i.d.}{\sim} N(0, \sigma_\mu)$. This shock is similar to that considered by Comin and Gertler (2006).[12]

The solution of the household problem yields the standard Euler equations for the risk-free interest rate and the cost of capital, and the relationship of wages to the marginal rate of substitution between consumption and labor:[13]

$$1 = E_t\left[\left(\frac{\beta C_t}{C_{t+1}}\right) r_t\right], \tag{18}$$

$$1 = E_t\left[\left(\frac{\beta C_t}{C_{t+1}}\right)(q_{t+1} + (1-\delta))\right], \tag{19}$$

$$W_t = \mu_t l_t^\psi C_t. \tag{20}$$

2.6 Equilibrium

A competitive equilibrium for this economy is a set of prices and allocations such that given prices, households, producers of final and intermediate goods and entrepreneurs solve their maximization problems and markets clear. The capital rental market clears when the demand for capital by intermediate-goods producers equals the supply by households. The labor market clears when firms' demand for labor equals labor supply by households. Finally, the final-goods market clears if production equals demand for consumption, capital accumulation and entrepreneurship:

$$Y_t = C_t + I_t + X_t. \tag{21}$$

In equilibrium, the change in the average productivity of the economy is given by the number of sectors that experience an innovation

$$A_t = \int_0^1 \left[n_{j,t-1}A_{t-1}^{\max} + (1-n_{j,t-1})A_{j,t-1}\right] dj = n_{t-1}\left(A_{t-1}^{\max} - A_{t-1}\right) + A_{t-1}, \tag{22}$$

which describes how productivity increases as a linear function of the distance to the technology frontier $A_{t-1}^{\max} - A_{t-1}$ and the entry rate of new firms n_{t-1} (the percentage of sectors in which a new incumbent appears).

The model has a deterministic steady-state solution that follows a balanced-growth path, in which variables Y_t, C_t, I_t, X_t, A_t, $A_t^{\max}$, W_t, Π_t, and K_t grow at a constant rate, whereas n_t, q_t, l_t, and g_t are constant. This constant growth rate is a function of the model's structural

12 In Comin and Gertler (2006), this shock reflects changes in the elasticity of substitution across different types of labor, whereas in our case it is a shock to the households' value of leisure. Notwithstanding, the effect on the model variables is the same. We have chosen this specification in light of the criticism of labor shocks by Chari et al. (2009).

13 In this model, the cost of capital is procyclical whereas in the data it is countercyclical. We thank an anonymous referee for this remark.

parameters.[14] To detrend the nonstationary variables, we divide them by $A_t^{\max}$. The complete set of equations is presented in Appendix A.1.

2.7 The efficient allocation

In the model presented above, the competitive equilibrium is not socially optimal. This is so because of monopolistic competition in the intermediate-goods sector and spillovers associated with decentralized innovation, which entrepreneurs do not internalize when making their R&D decisions. These distortions produce an inefficient allocation of resources, as discussed by Aghion and Howitt (1992). To see how the economy behaves in the Pareto-optimal case, we assume that the economy is managed by a benevolent social planner who maximizes

$$\max_{C_t, I_t, K_t, l_t, X_t, A_t, A_t^{\max}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \mu_t \frac{l_t^{1+\psi} - 1}{1+\psi} \right]$$

subject to the aggregate resource constraint (21), the capital accumulation law (17), the production function of entrepreneurs (10), aggregate productivity (22) and the spillover effect of innovation on technology growth (14). The complete set of equations is presented in Appendix A.2.

The decentralized economy differs from the efficient allocation in two respects. First, monopolistic competition in the intermediate sector induces underinvestment in physical capital, which is given by

$$E_t \left[\left(\beta \frac{C_t}{C_{t+1}} \right) \left(\alpha \frac{Y_{t+1}}{K_t} + (1-\delta) \right) \right] = 1, \tag{23}$$

instead of

$$E_t \left[\left(\beta \frac{C_t}{C_{t+1}} \right) \left(\alpha^2 \frac{Y_{t+1}}{K_t} + (1-\delta) \right) \right] = 1 \tag{24}$$

in the decentralized equilibrium (Eqs. 6 and 19). Second, in the decentralized economy, entrepreneurs cannot capture the full consumer surplus associated with the goods that they create and they do not internalize the effect of the redistribution of rents from past innovators to current innovators or the positive impact of their innovations on the technology frontier. In the first-best allocation, R&D investment is given by

$$E_t \left[\left(\beta \frac{C_t}{C_{t+1}} \right) \left(\Gamma_{1,t+1} n_t + \Gamma_{2,t+1} g_{t+1} \right) \right] = \frac{X_t}{A_t^{\max}} + \Gamma_{2,t}, \tag{25}$$

$$E_t \left[\left(\beta \frac{C_t}{C_{t+1}} \right) \Gamma_{1,t+1} (1-n_t) \right] = -\frac{(1-\alpha) Y_t}{A_t} + \Gamma_{1,t}, \tag{26}$$

$$E_t \left[\left(\beta \frac{C_t}{C_{t+1}} \right) \left(\Gamma_{1,t+1} \left(A_t^{\max} - A_t \right) + \Gamma_{2,t+1} \sigma_t A_t^{\max} \right) \right] = (1+\eta) \frac{X_t}{n_t}, \tag{27}$$

where $\Gamma_{1,t}$ and $\Gamma_{2,t}$ are the Lagrange multipliers of (22) and (14), respectively, whereas in the decentralized economy, it is given only by Eq. 13.

[14] The rate of growth in steady-state can be expressed as $g = 1 + \sigma n$, where n is the steady-state value of n_t.

3 Welfare analysis in the steady-state

This section analyzes the long-run implications of the model. We show that a pair of capital and R&D subsidies can correct all the market failures and restore the optimal steady-state. We also calibrate the model to match the average values of the most relevant macroeconomic variables in the US post-war period and compute welfare in the decentralized economy and the efficient allocation.

3.1 Optimal capital and R&D subsidies

Howitt and Aghion (1998) propose subsidizing capital costs and R&D investments as two complementary tools to improve the allocative efficiency of the economy. The cost of capital to intermediate producers can be subsidized at rate ϕ so that capital rents can be expressed as $q_t(1-\phi)K_{j,t-1}$ in Eq. 5. R&D investments can be subsidized at a rate τ so that entrepreneurs invest $(1-\tau)X_{j,t}$ in Eq. 12. Both subsidies can be financed by a non-distortionary lump-sum tax on households.

Proposition 1 (Optimal subsidies) *The steady-state allocation in the decentralized equilibrium may be made equal to the efficient allocation by introducing capital and R&D subsidies financed by a lump-sum tax on households. The optimal value of the capital subsidy is* $\phi^{opt} = 1-\alpha$ *and the optimal value of the R&D subsidy is*

$$\tau^{opt} = 1 - \frac{\alpha(1+\sigma)\left(\frac{(1+\eta)(1+\sigma n^*)}{\sigma\beta} + \frac{n^*}{(1-\beta)}\right)}{(\eta+1)\left(\frac{(1+\sigma n^*)}{\beta} + \frac{n^*(1+\sigma)}{(1-\beta)}\right)},$$

where n^* is the steady-state value of n in the efficient allocation, a function of the structural parameters of the model.

Proof See Appendix B. ■

This result guarantees that two instruments are sufficient to recover the first-best allocation in the steady-state. It should also be noted that this result applies only to the steady-state allocation. We leave out-of-the-steady-state results for the next section.

To analyze social welfare, we provide an algebraic expression for the steady-state household utility in the following lemma

Lemma 2 *Normalizing $A_0^{\max} = 1$, the steady-state value of the representative household's utility is*

$$U = \frac{\log(c)}{(1-\beta)} - \frac{\mu}{(1-\beta)}\frac{l^{1+\psi}-1}{(1+\psi)} + \frac{\beta\log(g)}{(1-\beta)^2}. \tag{28}$$

Proof See Appendix B. ■

Social welfare depends on steady-state effective consumption c, labor l and the *rate of growth of TFP* g. Growth is weighted by $\frac{\beta}{(1-\beta)^2}$, which is larger than the weight $\frac{1}{(1-\beta)}$ of consumption and labor as long as β is greater than 0.5. That is, as long as households are not too impatient, much greater weighting is given to growth (which implies future consumption) than to initial consumption and leisure.

Table 1 Calibration

Steady-state parameters								Shock parameters		
α	β	δ	ψ	μ	σ	η	$\log(\lambda)$	ρ_μ	σ_μ	σ_σ
0.65	0.98	0.10	1	1	0.19	11	23.1	0.5	0.05	3

In the "steady-state optimal subsidy", the value of subsidies is ($\phi = 0.65$, $\tau = 0.69$)

3.2 Calibration

The parameters of the model are calibrated to match key empirical evidence for the US economy in the post-war period 1950–2007. The complete list of parameters can be found in Table 1. Information about data sources is provided in Appendix D.

RBC parameters. The first set of parameters is common to most of the RBC literature. The value of α is set to 0.35 so that the share of output devoted to labor compensation is 65%. As commented in Sect. 2, we abstract from population growth by assuming that variables are scaled by the working-age population. The average growth of gross domestic product (GDP) per working-age person $\log(g)$ is 1.9% so that assuming a real interest rate $\log(r)$ of 4%, we obtain a value of $\beta = \frac{g}{r} \approx \frac{1}{1.02}$. Capital depreciation δ is set to 10% per year. Thus, the capital investment share of GDP results in $i/y = \alpha^2(g - (1-\delta))/(r - (1-\delta)) = 10.5\%$, close to the 12.8% for the post-war period. We calibrate μ to roughly match the product of employment and working hours. Average employment was 94% and average hours worked per 40-h week was 88%, so that the product is 83%. We set μ to 1 so that l is 86%. We set the Frisch labor supply elasticity $1/\psi$ to unity, as in Christiano et al. (2005) and in Comin and Gertler (2006).

Endogenous growth parameters. These parameters characterize the technology spillover σ and the relationship between R&D investment and business turnover in Eq. 10: λ, η. The average business turnover of US firms n (the rate of creation/destruction of firms in the economy) in the last two decades has been 10%. As shown in Fig. 1, this value is also consistent with the empirical evidence for average survival rates in the 1963 and 1976 cohorts of US manufacturing firms. Given this value for n, to replicate the value of GDP growth of 1.9%, we set the spillover coefficient σ to 0.19 as $\sigma = (g-1)/n \approx \log(g)/n$.

We calibrate R&D spending x/y as the average of non-federal-supported R&D investment in the last three decades, namely 1.6% of GDP. Given this value we may set $\eta = \frac{1}{(x/y)}\frac{\alpha n(1-\alpha)(1+\sigma)}{(r+n-1)} - 1$ to 11. Finally, we set $\log(\lambda)$ to 23.1 to match the value of $n = 10\%$. The main results of the paper are quite robust to different parameter values.[15]

3.3 Numerical results

Table 2 compares the US historical averages with the calibrated steady-state model values ("benchmark") of some of the main aggregate variables. Additionally, we compute the counterfactual efficient allocation and the cases of introducing capital and R&D subsidies.

To compute the efficient allocation, we solve the social planner's problem for the set of calibrated parameters presented above. In this case, total R&D investment is 5.4% of GDP, which results in increases in long-term growth (2.2%) and business turnover (11.7%). Capital investment amounts to 30% of GDP in this case. The ratio of R&D investment in the

15 Such as calibrating the R&D share x/y to 2.6% (post-war total R&D) or n to 3.6% as in Howitt (2000). In Nuño (2010) we provide some robustness tests for different parameter values.

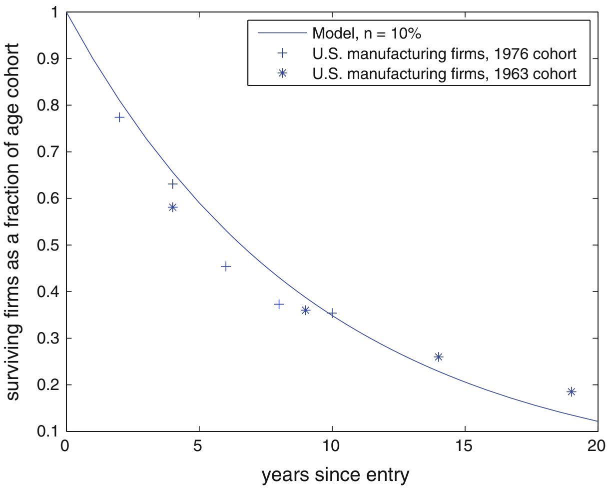

Fig. 1 Survival rates of firms in the US economy and in the model $(1 - n)^t$

Table 2 Steady-state analysis

%	Welfare	GDP growth	Turnover	Investment	R&D
Data	–	1.9	10.0	12.8	1.6
Benchmark	0	1.9	10.0	10.5	1.6
Efficient allocation	53.1	2.2	11.7	30.0	5.4
Subsidies					
Capital ($\phi = 0.65$)	47.2	2.0	10.6	29.9	1.6
R&D ($\tau = 0.69$)	5.4	2.1	11.0	10.5	5.3
Optimal ($\phi = 0.65$, $\tau = 0.69$)	53.1	2.2	11.7	30.0	5.4

Welfare refers to per capita consumption gains with respect to the benchmark
GDP growth refers to the growth of GDP per working-age person
Investment refers to capital investment

efficient allocation to that in the decentralized equilibrium is 3.3. This result seems to be in line with the previous literature. Estimating the social return to R&D, Jones and Williams (1998) conclude that a "conservative" lower bound to the ratio between optimal R&D investment and actual investment would be around 4. In a calibrated model, Jones and Williams (2000) find ratios between 1 and 3 for plausible parameterizations.

We use Lemma 2 to compute social welfare. In the spirit of Lucas (1987), we express welfare gains as changes in annual per capita consumption with respect to the decentralized economy. That is, we compute the value $\varkappa$ so that the steady-state utility of the decentralized economy U^{DE} with augmented consumption $U((1+\varkappa)c; l) = U^{DE}(c; l) + \log(1+\varkappa)/(1-\beta)$ is equal to the utility of the counterfactual economy $U^*(c^*; l^*)$.[16] Our results show that the efficient allocation generates a welfare gain equivalent to 53% of annual consumption with respect to the decentralized equilibrium.

To mitigate the welfare loss due to the suboptimal allocation of resources in the decentralized equilibrium, governments may subsidize capital and R&D expenditures so that

[16] Lucas (1987) considers that households' utility depends only on consumption.

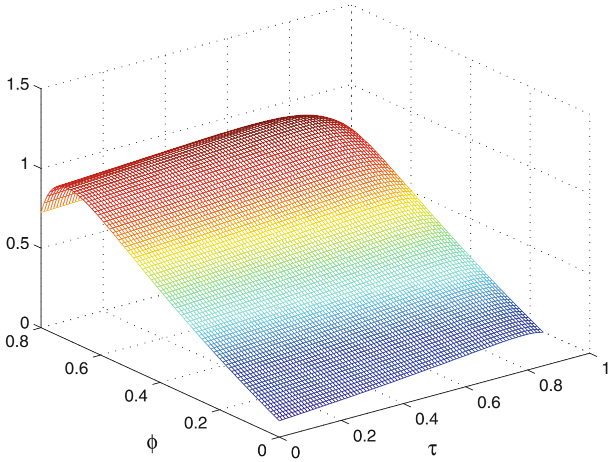

Fig. 2 Welfare as a function of capital (ϕ) and R&D (τ) subsidies

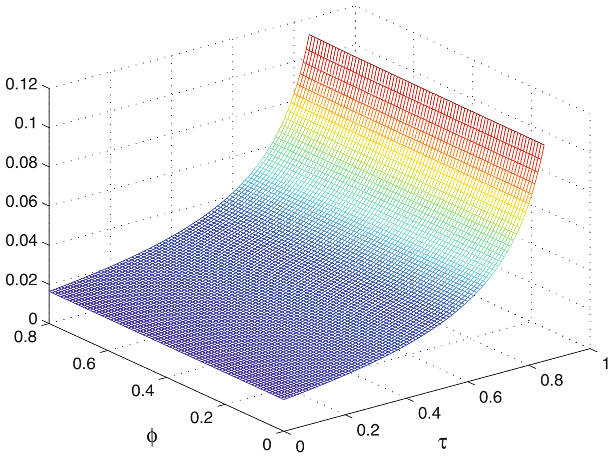

Fig. 3 R&D investment over GDP as a function of capital (ϕ) and R&D (τ) subsidies

innovators internalize some of the externalities derived from their activities.[17] In Figs. 2 and 3, we explore how different combinations of capital and R&D subsidies affect social welfare and the R&D investment share. These figures show that capital subsidies have a higher impact on welfare whereas R&D subsidies affect more significantly the level of R&D investment.

Table 2 quantifies the impact of subsidies in three distinct cases. First, we compute the values for the optimal combination of capital and R&D subsidies ($\phi = 0.65$, $\tau = 0.69$) which, according to Proposition 1, is sufficient to restore the efficient allocation. We then analyze the impact of each of these subsidies independently. Results show that both subsidies are needed to restore the first-best allocation, although most of the welfare losses in the

[17] Although the paper focuses on capital and R&D subsidies, other policy interventions are possible. We leave this important issue for further research.

decentralized equilibrium are the result of suboptimal capital investment due to imperfect competition in the intermediate-goods sector.

The result that most welfare losses are due to monopolistic competition could be the consequence of having chosen a counterfactually large mark-up. In our model, as in Howitt and Aghion (1998), the mark-up is $1/\alpha$, which results in 185% for a value of α of 0.35. In contrast, Jones and Williams (2000) consider a mark-up around 30%. To explore how the value of the mark-up affects the results, we recalibrate the model, following Kwan and Lai (2003), to replicate the 30% mark-up instead of the labor share of 65%. We set $\alpha = 1/1.3,\ \eta = 8, \log(\lambda) = 21$ and maintain the values of the other parameters as in the benchmark calibration. In this case, the welfare gain in the efficient allocation is 86% of annual consumption. By subsidizing capital ($\phi = 1 - 1/1.3$) we achieve a welfare gain of 85%, whereas by subsidizing R&D ($\tau = 0.30$) we only achieve 1%. This means that in the case with realistic mark-ups, even more of the welfare losses are due to monopolistic competition. Notwithstanding, this is not a general result, and it could be the case that in a model *à la* Jones and Williams (2000) with an extra degree of freedom to match both the mark-up *and* the labor share, welfare losses due to R&D distortions may be larger than those due to monopolistic competition.

4 Dynamic welfare analysis

This section explores the welfare implications of inefficiencies in endogenous growth in a dynamic stochastic context. To this end, we calibrate the stochastic processes of the exogenous shocks in an attempt to replicate the macrodynamics of GDP and R&D in the US economy. We then compare the dynamic responses of the model with those of the counterfactual efficient allocation and discuss the dynamic effects of different policy interventions. Finally, we compute the social welfare in a dynamic context.

4.1 Calibration of the shocks

Comin and Gertler (2006) and Barlevy (2007) have documented that private R&D expenditures have been procyclical during the post-war period. Barlevy finds a positive correlation between the growth rate of R&D expenditure and GDP growth of 0.39, and Comin and Gertler recompute this correlation in per capita terms and find it to be 0.31 for the medium-term cycle and 0.3 for the high-frequency component. We compute the correlation between both growth rates in per capita terms without any filtering and find it to be 0.27. We consider the working-age population (aged 16–65), following Comin and Gertler (2006). We present this value together with the standard deviations of growth rates of GDP and R&D and the first element of the autocorrelation of GDP growth in the first row of Table 3.

We calibrate the three parameters ($\rho_\mu,\ \sigma_\mu,\ \sigma_\sigma$) of the two shocks considered, μ_t and σ_t, to replicate the three moments of interest: the volatility of GDP growth, the autocorrelation of GDP growth and the correlation between R&D and GDP.[18] Parameter values are presented

[18] We have chosen the labor shock μ_t instead of a temporary productivity shock to make our results more comparable to those of Comin and Gertler (2006). Notwithstanding, in this model the impulse responses of both shocks are the same, as we show in Nuño (2010), so results do not change if instead we consider an aggregate total factor productivity shock.

Table 3 Second-order moments of GDP and R&D growth

%	Volatilities		Autocorrelation	Correlation
	GDP	R&D	GDP	GDP, R&D
Data	2.2	4.4	8.8	26.6
Benchmark	2.4	3.3	8.6	25.6
Efficient allocation	2.9	6.4	9.3	−8.2
Steady-state optimal subsidies	2.9	4.2	9.1	2.8
R&D countercyclical subsidy	2.4	9.1	8.9	−8.1

All moments refer to the GDP and R&D growth per working age person
The "autocorrelation" is the first element of the autocorrelation vector

in Table 1.[19] We also show how the volatility of R&D growth, 3.3%, is close to the value in the data for the post-war period, 4.4%.[20]

The model is able to replicate the procyclicality of R&D due to the "lab-equipment" specification of the R&D process: the opportunity cost of R&D is the price of the final good, which is constant over time, whereas the value of a successful innovation moves procyclically with the profits and the aggregate output, as discussed in Aghion and Saint-Paul (1998). Hence, decentralized R&D investment rises during expansions and falls during recessions.

We should remark that R&D procyclicality is not a consequence of having included endogenous labor supply in the model, in contrast to Fatas (2000). In our model, endogenous labor supply plays a minor role in the procyclicality of R&D. To check this, we recompute the model assuming inelastic labor supply, calibrated to maintain the same steady-state as in the benchmark. In this case, instead of a labor shock we have considered a temporary aggregate TFP shock. The procyclicality of R&D does not change with inelastic labor. The impact of variable labor supply would be greater if R&D depended only on labor, and not on final goods. However, Barlevy (2007) states that, even in this case, variable labor supply is not sufficient to generate procyclicality if changes in the opportunity cost of R&D over the cycle are taken into account.

4.2 Counterfactual second-order moments

Table 3 also presents the calculations of the second order moments for the efficient allocation and two policy interventions. We find that, in the Pareto-optimal allocation, R&D is slightly countercyclical: the correlation coefficient is -8%. This result confirms the finding of Barlevy (2007) that the optimal allocation of R&D is more countercyclical than that in the decentralized equilibrium. We also find that output is more volatile and persistent in the efficient allocation than in the decentralized economy. Figures 4 and 5 show the impulse responses to labor and a knowledge-spillover shocks, respectively. We display results for GDP Y_t, R&D share $\frac{X_t}{Y_t}$, business turnover n_t and labor supply l_t. Since GDP Y_t is nonstationary, we display the deviations of GDP with respect to its log-linear long-run trend $g = 1 + \sigma n$.[21] The labor

[19] Computations are performed in Dynare. The data frequency is annual. We always refer to the GDP and R&D expenditure per person aged 15–64. Additional information about data sources may be found in Appendix D.

[20] Instead of calibrating the parameters of these two shocks, we could have estimated them, with similar results. In Nuño (2010), we display some robustness tests to alternative specifications of shocks.

[21] We plot $\tilde{y}_t \equiv \sum_{i=0}^{t-1} \left[\log\left(\frac{Y_{i+1}}{Y_i}\right) - \log(1+\sigma n) \right] = \sum_{i=0}^{t-1} \log\left(\frac{y_{i+1} g_i}{y_i (1+\sigma n)}\right)$.

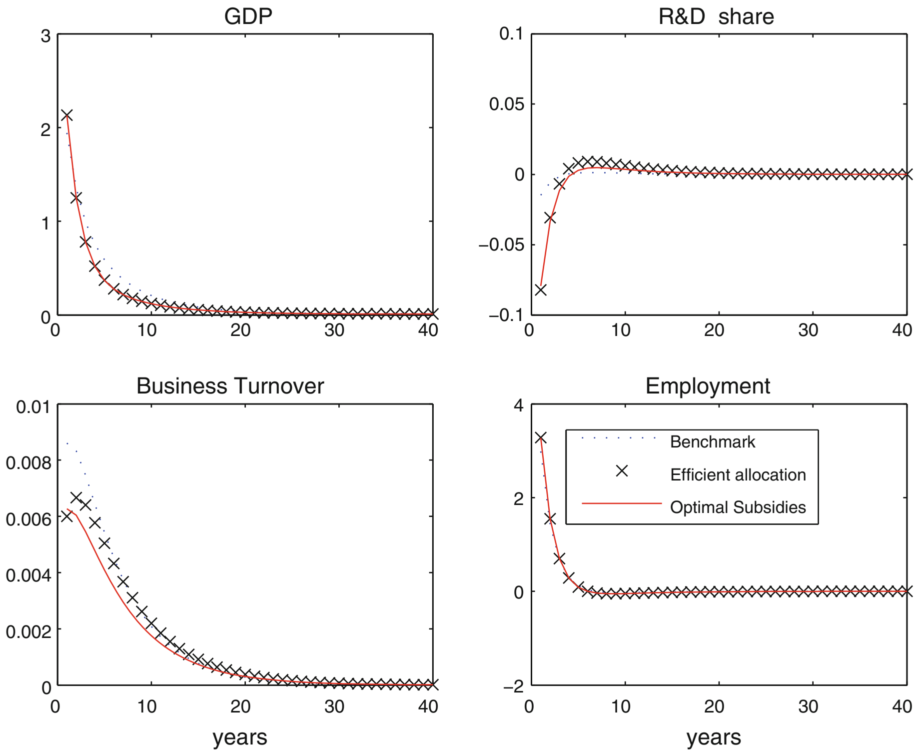

Fig. 4 Impulse responses to a labor shock μ_t

shock generates a temporary response on output, whereas the spillover shock generates a persistent effect. In both cases, the magnitude of the responses of output, labor supply and the R&D share are larger in the optimal allocation than the responses in the benchmark decentralized equilibrium.

We also analyze the response of the economy in the case of the optimal steady-state subsidies; i.e., when the two subsidies presented in Proposition 1 are introduced. Although in this case the steady-state in the decentralized equilibrium is the same as the Pareto-optimal steady-state, its dynamic response differs; R&D is almost acyclical (correlation of 2.8%) and less volatile than in the efficient allocation. This implies that even in this case where there are no steady-state distortions, dynamic distortions still play a role that, in principle, makes the decentralized equilibrium suboptimal. Notwithstanding, Figs. 4 and 5 show how the impulse responses of output and labor are indistinguishable from the optimal ones. It is R&D and business turnover that have different responses.

Finally, we examine the performance of a countercyclical R&D subsidy. In this case, the constant R&D subsidy τ is replaced by a time-varying one τ_t given by

$$\tau_t = -\kappa \log\left(\frac{y_t}{y_{t-1}}\right); \tag{29}$$

that is to say, the government countercyclically modifies the subsidy rate with a slope κ as a function of the dynamics of output $y_t \equiv \frac{Y_t}{A_t^{\max}}$. The steady-state value of this subsidy is zero, and thus the subsidy does not alter the steady-state properties of the decentralized economy,

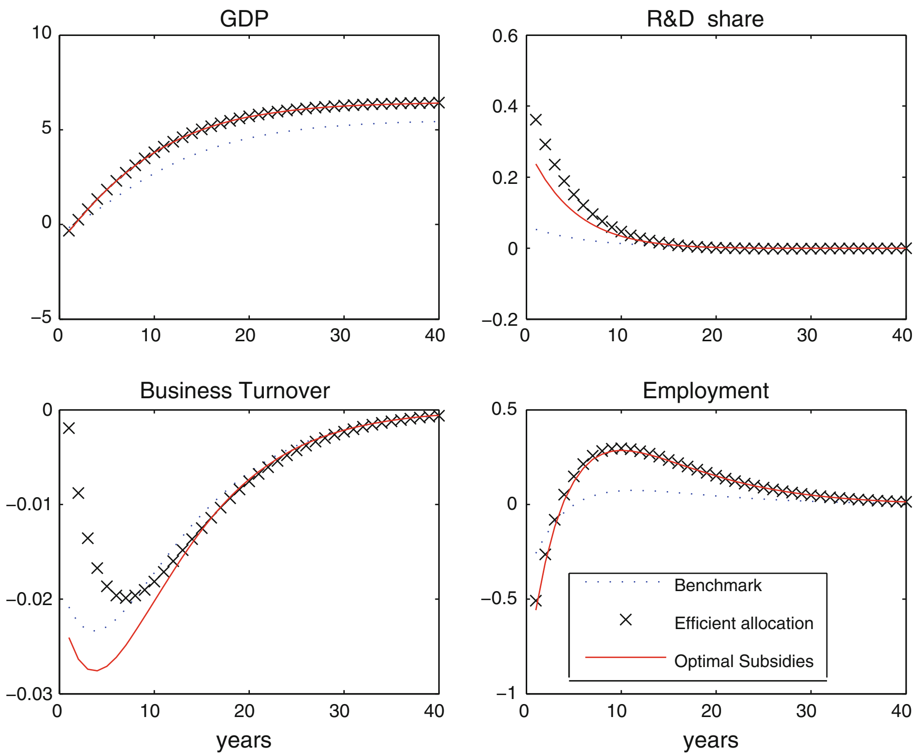

Fig. 5 Impulse responses to a spillover shock σ_t

only its dynamics. We set $\kappa = 6.2$ to replicate the value of the correlation of R&D and GDP in the efficient allocation. Results show how the remaining second-order moments under consideration differ from those in the Pareto-optimum.

4.3 Welfare analysis

To analyze the welfare effects of the procyclicality of R&D, we need to compute the social welfare of the model under stochastic shocks. To this end, we propose in Appendix C an algorithm that allows the numerical computation of the unconditional expected utility function of households by Monte Carlo simulation. The advantage of this algorithm is that it works both with stationary and nonstationary variables and that it does not require any algebraic manipulation of the utility function. We express welfare as the gain (or loss) in annual per capita consumption with respect to the decentralized economy without shocks. The Monte Carlo method employs 10,000 simulations of 500 periods of the model. We check that the approximation error introduced by the algorithm is smaller than $\pm 0.25\%$ of annual consumption.

The first line of Table 4 shows 'steady-state welfare'. The values are obtained by simulation, but they are the same as the theoretical values presented in the first column of Table 2. This implies that households would be indifferent between living in the Pareto-optimal economy without shocks and living in the benchmark economy (also without shocks) with an increase of 53% in their annual consumption.

Table 4 Dynamic welfare analysis

%	Benchmark	Efficient allocation	SS optimal subsidies	R&D policy rule
Steady-state welfare	0.0	53.1	53.1	0.0
Dynamic welfare	−8.5	35.4	35.4	−8.5
Cost of business cycles	8.5	11.5	11.5	8.5

Mean values after 10,000 Monte Carlo simulations of length 500 years

The second line shows 'dynamic welfare', which is defined as the share $\hat{\varkappa}$ of annual consumption that a household in the non-stochastic decentralized economy with expected utility U^{DE} would need to receive in order to be indifferent between remaining in its situation and accepting living in a world with aggregate fluctuations and expected utility $\tilde{U}$, $\hat{\varkappa} = \exp((1-\beta)(\tilde{U} - U^{DE})) - 1$. For example, a household would demand an increase in its consumption of 35% to remain in the non-stochastic decentralized economy instead of living in the first-best economy with aggregate fluctuations. The same household would be willing to *forgo* 8.5% of its consumption to avoid living in the decentralized economy with aggregate shocks.

We can check how the introduction of aggregate fluctuations reduces the welfare distance between the decentralized economy and the Pareto-optimal allocation. If macroeconomic shocks were to be introduced in the non-stochastic decentralized economy, households should receive a ex-ante compensation of 47% of their consumption to remain in the (now stochastic) decentralized economy instead of moving to the first-best allocation with fluctuations ($(1 + 0.35)/(1 - 0.085) = 0.47$). In contrast, in the non-stochastic case a household needs to be compensated by as much as 53% of its consumption to remain in the decentralized economy instead of moving to the first-best allocation.

A related concept is the cost of business cycles, which is defined as the share of the annual consumption $\tilde{\varkappa}$ that a household living in a non-stochastic economy with expected utility U^* should receive to be indifferent to living with aggregate fluctuations and expected utility $\tilde{U}$. In each case, the cost is expressed as a share of its steady-state consumption; i.e., $\tilde{\varkappa} = \exp((1-\beta)(\tilde{U} - U^*)) - 1$. It thus coincides with dynamic welfare in the case of the decentralized equilibrium (i.e. $U^* = U^{DE} \Rightarrow \tilde{\varkappa} = \hat{\varkappa}$) but not in the case of the efficient allocation or the subsidized economies. The third line of Table 4 shows how the cost of business cycles is higher in the efficient allocation than in the decentralized economy. A household in the efficient allocation should be compensated with 11.5% of its consumption for the introduction of macroeconomic shocks, whereas this reduces to 8.5% in the decentralized equilibrium. Therefore, the dynamic distortions in the decentralized equilibrium partially mitigate the suboptimality of the steady state, reducing the cost of business cycles instead of increasing it.

Finally, the last column of Table 4 shows that the countercyclical R&D subsidy fails to produce any positive increase in welfare. In contrast, the welfare in the case of the fiscal policy that restores the steady-state is very close to the optimum, despite the fact that the equilibrium is not fully efficient. The reason is that it replicates the optimal impulse responses of consumption and labor (as shown in Figs. 4 and 5), which are the two variables that determine utility.

5 The role of knowledge spillovers

The main conclusions about the welfare effect of business cycles discussed above are robust to different reasonable specifications of the calibrated parameters.[22] However, they are all based on a model where long-run productivity growth is an endogenous function of R&D investment as shown in Eq. 14. This endogeneity of the technology frontier has been assumed in order to follow as closely as possible the Howitt and Aghion (1998) model. Notwithstanding, there are reasons to believe that it may be at odds with reality. Several authors, such as Porter and Stern (2000), argue that the case $\sigma \approx 0$ (exogenous technology frontier) is empirically relevant at the level of a given country. This assumption can be considered as a limit case of Howitt (2000) where all countries are small with respect to the world economy. Along a different line, Comin (2004) argues that R&D plays a small role in determining long-run productivity growth.

In this section we analyze whether our results are robust to diverse specifications of long-run growth. Hence we extend Eq. 14 to include both endogenous and exogenous growth:

$$g_t \equiv \frac{A_t^{\max}}{A_{t-1}^{\max}} = 1 + \chi_t + \sigma_t n_{t-1}, \tag{30}$$

with $\log(\chi_t) = \log(\chi) + \varepsilon_t^{\sigma}$. In this case, the long-run growth rate is given by a constant χ plus an endogenous term σn.

We consider three extreme cases; (1) the fully endogenous case, with $\sigma = 0.19$ and $\chi = 0$, which was already analyzed, (2) the fully exogenous case, with $\sigma = 0$ and $\chi = 0.019$, where long-run growth does not depend at all on R&D, and (3) an intermediate case with $\sigma = 0.04$ and $\chi = 0.015$, where there are no R&D distortions in the steady-state. The remaining parameters have the values listed in Table 1. In this manner, we set the long-run properties of the model without modifying the dynamics. We should remark that, even in the case of a fully exogenous technology frontier, R&D plays a vital role in the economy as it allows firms to incorporate the latest technological innovations into their production process.

The magnitude of knowledge spillovers determines whether in a decentralized economy there is over- or under-investment in R&D. In addition to these spillovers, entrepreneurs face two market failures. On the one hand, they cannot appropriate the entire consumer surplus produced by a future innovation, which tends to generate under-investment. On the other hand, they do not internalize the profit loss of previous incumbents associated with creative destruction, which tends to generate over-investment. In an economy with large knowledge spillovers ($\sigma = 0.19$), there is under-investment in R&D, as shown in Table 5. However, in an economy with no spillovers ($\sigma = 0$), there is over-investment, as the optimal share of R&D is 0.7%. In the case of $\sigma = 0.04$ we guarantee that the three effects are equivalent, and hence the optimal R&D level coincides with that observed in the decentralized economy. In this case, the only distortion in the steady-state is the consequence of the monopolistic competition in the intermediate-goods sector.

We complement Proposition 1 by including also the case of no spillovers ($\sigma = 0$). The value of the optimal steady-state subsidies is given by Proposition 3.

Proposition 3 (Optimal subsidies in the case of no knowledge spillovers) *The steady-state allocation in the decentralized economy without knowledge spillovers may be made equal to the efficient allocation by introducing capital and R&D subsidies financed by a lump-sum*

[22] Some of the robustness checks can be found in Nuño (2010), where we employ a different parameterization and obtain similar qualitative results.

Table 5 Analysis of different assumptions about knowledge spillovers

%	Data	Endogenous	Exogenous	Mixed
	–	$\sigma = 0.19$	$\sigma = 0$	$\sigma = 0.04$
R&D in steady-state				
Decentralized economy	1.6	1.6	1.6	1.6
Efficient allocation	–	5.4	0.7	1.6
Capital subsidy ($\phi = 0.65$)	–	1.6	1.6	1.6
Correlation GDP, R&D				
Decentralized economy	26.6	26.0	26.0	26.0
Efficient allocation	–	−8.2	−18.0	−15.5
Capital subsidy ($\phi = 0.65$)	–	4.0	5.6	5.5
Steady-state welfare				
Decentralized economy	–	0.0	0.0	0.0
Efficient allocation	–	53.1	41.4	42.3
Capital subsidy ($\phi = 0.65$)	–	47.2	41.0	42.3
Dynamic welfare				
Decentralized economy	–	−8.5	−8.5	−8.5
Efficient allocation	–	35.4	28.9	29.4
Capital subsidy ($\phi = 0.65$)	–	33.0	28.8	29.4
R&D rule ($\kappa = 6.2$)	–	−8.5	−8.5	−8.5
Cost of business cycles				
Decentralized economy	–	8.5	8.5	8.5
Efficient allocation	–	11.5	8.8	9.1
Capital subsidy ($\phi = 0.65$)	–	9.6	8.6	9.1
R&D rule ($\kappa = 6.2$)	–	8.5	8.5	8.5

tax on households. The optimal value of the capital subsidy ϕ^{opt} *is* $1 - \alpha$ *and the optimal value of the R&D subsidy* τ^{opt} *is*

$$\tau^{opt} = 1 - \frac{\alpha(n^* + \chi)}{\chi},$$

where n^* *is the steady-state value of* n *in the efficient allocation.*

Proof See Appendix B. ■

In this case the value of the optimal capital subsidy is the same as in the case with spillovers ($\phi = 0.65$). However, the optimal subsidy is $\tau = -1.17$; i.e., the government should tax (instead of subsidizing) R&D activities to reduce innovation and business turnover. In the mixed case ($\sigma = 0.04$), the optimal capital subsidy is still $\phi^{opt} = 1 - \alpha$ whereas the optimal R&D subsidy is zero, as there are no R&D distortions in the steady-state.

Table 5 shows the main results regarding the cyclical properties of R&D and the welfare analysis in the three cases. It should be noted that, despite the differences in the optimal steady-state allocation of R&D resources, the dynamic and welfare conclusions are quite robust. Optimal R&D is always countercyclical despite the fact the decentralized R&D is procyclical. The welfare difference between the efficient allocation and the decentralized economy is reduced in the presence of business cycles, so that a household living in the non-stochastic decentralized economy will value more ex-ante the Pareto-optimum in the absence of shocks than in the stochastic case. Conversely, the cost of business cycles is higher in the efficient allocation than in the decentralized economy. Additionally, we analyze what happens with a fiscal intervention that eliminates the distortion associated with monopolistic competition. Again, in the three cases this policy significantly improves welfare, although the economy is still suboptimal due to market failures in R&D. Finally, we confirm how countercyclical R&D policies are not welfare-improving, independently of the value of σ.

The explanation of these results is that, regardless of whether there is under- or over-investment in R&D, as long as there are several market failures in the economy, the emergence of additional dynamic distortions due to the inefficient allocation of R&D resources over the business cycle leads to a second-best situation where some of the steady-state distortions are mitigated. The consequence is that the cost of business cycles is lower in the decentralized economy and that policy intervention aimed at correcting only this dynamic R&D inefficiency are not welfare-improving, whereas fiscal policies that correct steady-state distortions significantly improve social welfare.

6 Conclusions

This paper examines the cost of business cycles in an economy with endogenous Schumpeterian growth. The paper confirms the result in Barlevy (2007) that, although R&D is procyclical in a decentralized economy, the optimal path of R&D is countercyclical. However, this result does not necessarily imply that procyclical R&D increases the welfare cost of macroeconomic shocks and that countercyclical R&D subsidies are welfare-improving. On the contrary, we show how business cycles are more costly in the first-best allocation than in the decentralized economy and that countercyclical R&D policies have a null effect on welfare.

The reason behind these results is that the steady-state in a decentralized economy is distorted due to a number of market failures and the suboptimal propagation of aggregate shocks mitigates some of the costs associated with these market failures. In this case, a policy intervention that addresses only one of the distortions, as in the case of a countercyclical R&D subsidy, may have no positive effects on welfare, as stated by Lipsey and Lancaster (1956) in the theory of the second-best.

We also show that, if the first-best steady-state allocation is known to policymakers, it is possible to introduce in the decentralized economy a couple of subsidies, one to capital accumulation and the other to R&D, financed by lump-sum taxes on households, which restore the optimal steady-state. In this case, the welfare in the decentralized economy with aggregate fluctuations is close to the optimum.

It is important to analyze whether these conclusions still hold in a model with a more realistic tax structure. If no lump-sum taxes are possible, the presence of distortionary taxes could potentially modify some of the results. Additionally, a more detailed modeling of the R&D process, where incumbents also perform R&D and firms face credit frictions as in Aghion et al. (2005), or endogenous implementation cycles as in Francois and Lloyd-Ellis (2009), would enrich the analysis. We leave these issues for future research.

Acknowledgements I would like to thank Jim Costain, Juan Francisco Jimeno, Alessio Moro, Anton Nakov, Margarita Rubio, Pedro Teles, Carlos Thomas, Francesca Viani and four anonymous referees for their comments, as well as seminar participants at the 2008 EEA meeting in Milano, the 2009 CEMLA Conference in Salvador de Bahia, the 2010 Dynare Conference in Tuusula and a seminar at the Bank of Portugal. I am also grateful to the Department of Economics of the University of Cagliari, as part of this paper was written during a short stay there. The views expressed are those of the author and do not represent the views of the Banco de España or the Eurosystem.

Appendix A: complete set of equations

A.1 Market economy

We define a variable $s_t \equiv S_t/A_t^{\max}$ as the detrended version of variable S_t. The complete set of stationary equilibrium equations is as follows

Firms and aggregate production

$$w_t = (1-\alpha)\frac{y_t}{l_t}, \tag{31}$$

$$y_t = \left(\frac{k_{t-1}}{g_t}\right)^{\alpha}(a_t l_t)^{1-\alpha}, \tag{32}$$

$$q_t = \alpha^2\frac{y_t g_t}{k_{t-1}}, \tag{33}$$

Households and capital accumulation

$$1 = E_t\left[\left(\frac{\beta}{g_{t+1}}\frac{c_t}{c_{t+1}}\right)r_t\right], \tag{34}$$

$$1 = E_t\left[\left(\frac{\beta}{g_{t+1}}\frac{c_t}{c_{t+1}}\right)(q_{t+1}+(1-\delta))\right], \tag{35}$$

$$w_t = \mu_t l_t^{\psi} c_t, \tag{36}$$

$$k_t = i_t + \frac{(1-\delta)k_{t-1}}{g_t}, \tag{37}$$

Entrepreneurs and TFP

$$x_t = \lambda n_{j,t}^{(\eta+1)}, \tag{38}$$

$$x_t = \frac{n_t E_t\left[v_{t+1}\right]}{(\eta+1)r_t} \tag{39}$$

$$v_t = \alpha(1-\alpha)\frac{y_t}{a_t} + E_t\left[\frac{(1-n_t)}{r_t}v_{t+1}\right], \tag{40}$$

$$a_t g_t = n_{t-1}(1-a_{t-1}) + a_{t-1}, \tag{41}$$

$$g_t = 1 + \sigma_t n_{t-1}, \tag{42}$$

Aggregate budget constraint

$$y_t = c_t + i_t + x_t, \tag{43}$$

with a vector of variables $[y_t, c_t, i_t, x_t, n_t, k_t, a_t, r_t, w_t, q_t, g_t, l_t, v_t]$.

A.2 Efficient allocation

$$(1-\alpha)y_t = \mu_t l_t^{\psi+1} c_t, \tag{44}$$

$$y_t = \left(\frac{k_{t-1}}{g_t}\right)^{\alpha}(a_t l_t)^{1-\alpha}, \tag{45}$$

$$k_t = i_t + \frac{(1-\delta)k_{t-1}}{g_t}, \tag{46}$$

$$x_t = \lambda n_{j,t}^{(\eta+1)}, \tag{47}$$

$$a_t g_t = n_{t-1}(1 - a_{t-1}) + a_{t-1}, \tag{48}$$

$$g_t = 1 + \sigma_t n_{t-1}, \tag{49}$$

$$y_t = c_t + i_t + x_t, \tag{50}$$

$$E_t\left[\left(\frac{\beta}{g_{t+1}}\frac{c_t}{c_{t+1}}\right)\left(\gamma_{1,t+1}n_t + \gamma_{2,t+1}g_{t+1}\right)\right] = x_t + \gamma_{2,t}, \tag{51}$$

$$E_t\left[\left(\frac{\beta}{g_{t+1}}\frac{c_t}{c_{t+1}}\right)\gamma_{1,t+1}(1 - n_t)\right] = -\frac{(1-\alpha)y_t}{a_t} + \gamma_{1,t}, \tag{52}$$

$$E_t\left[\left(\frac{\beta}{g_{t+1}}\frac{c_t}{c_{t+1}}\right)\left(\gamma_{1,t+1}(1 - a_t) + \gamma_{2,t+1}\sigma_t\right)\right] = (1+\eta)\frac{x_t}{n_t}, \tag{53}$$

$$E_t\left[\left(\frac{\beta}{g_{t+1}}\frac{c_t}{c_{t+1}}\right)\left(\alpha\frac{y_{t+1}g_{t+1}}{k_t} + (1-\delta)\right)\right] = 1, \tag{54}$$

with a vector of variables $\left[y_t, c_t, i_t, x_t, n_t, k_t, a_t, g_t, l_t, \gamma_{1,t}, \gamma_{2,t}\right]$, where $\gamma_{1,t}$ and $\gamma_{2,t}$ are the stationary Lagrange multipliers of (22) and (14), respectively.

Appendix B: proofs

Proposition 1

Proof To demonstrate the proposition, we show that there are two constants ϕ^{opt} and τ^{opt} such that the steady-state system of equations in the case of the decentralized economy with subsidies is the same as the system of equations in the efficient allocation.

For the market economy, Eqs. 6 and 19 can be combined as

$$\left(\frac{\beta}{g}\right)\left(\alpha^2\frac{yg}{k(1-\phi)} + (1-\delta)\right) = 1,$$

whereas the equivalent equation for the efficient allocation is

$$\left(\frac{\beta}{g}\right)\left(\alpha\frac{yg}{k} + (1-\delta)\right) = 1,$$

therefore, to recover the first-best allocation $\frac{\alpha^2}{(1-\phi^{opt})} = \alpha$ and $\phi^{opt} = 1 - \alpha$.

The Lagrange multipliers and R&D investment in the efficient allocation are

$$\gamma_1 = \frac{(1-\alpha)y\frac{g}{\beta}}{a\left(\frac{g}{\beta} + n - 1\right)},$$

$$\gamma_2 = \frac{(1-\alpha)yn}{a\left(\frac{g}{\beta} + n - 1\right)(1-\beta)} - \frac{x}{(1-\beta)},$$

$$x = \frac{(1-\alpha)y}{\left(\frac{g}{\beta} + n - 1\right)}\left(\frac{g}{\beta} + \frac{n}{a(1-\beta)}\right)\frac{1}{\left(\frac{(1+\eta)g}{\sigma n\beta} + \frac{1}{(1-\beta)}\right)}$$

In the decentralized economy, R&D investment is

$$x = \frac{n}{(1-\tau)(\eta+1)}\frac{\alpha(1-\alpha)y(1+\sigma)}{(r+n-1)},$$

and therefore, the optimal subsidy is

$$\tau^{opt} = 1 - \frac{n\alpha(1+\sigma)\left(\frac{(1+\eta)g}{\sigma n\beta} + \frac{1}{(1-\beta)}\right)}{\left(\frac{g}{\beta} + \frac{n}{a(1-\beta)}\right)(\eta+1)}.$$

The remaining equations are the same for the efficient allocation and the decentralized economy, and thus both systems of equations produce the same steady-state solutions. ■

Lemma 2

Proof Given Eq. 15, a household's expected utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - \mu_t \frac{l_t^{1+\psi} - 1}{1+\psi}\right]$$
$$= E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \log(A_{t-1}^{\max}) + \log(g_t) - \mu_t \frac{l_t^{1+\psi} - 1}{1+\psi}\right].$$

In the steady-state with $A_0^{\max} = 1$, this expression can be simplified to

$$\sum_{t=0}^{\infty} \beta^t \left[\log(c) + \log(A_0^{\max}) + \log(g^t) - \mu \frac{l^{1+\psi} - 1}{1+\psi}\right]$$
$$= \left[\log(c) - \mu \frac{l^{1+\psi} - 1}{1+\psi}\right]\left(\sum_{t=0}^{\infty} \beta^t\right) + \log(g)\left(\sum_{t=0}^{\infty} t\beta^t\right)$$
$$= \frac{\left[\log(c) - \mu \frac{l^{1+\psi}-1}{1+\psi}\right]}{1-\beta} + \frac{\log(g)\beta}{(1-\beta)^2}.$$

■

Proposition 3

Proof As in Proposition 1, we show that there are two constants ϕ^{opt} and τ^{opt} such that the steady-state system of equations in the case of the decentralized economy with subsidies is the same as the system of equations in the efficient allocation.

The reasoning for the optimal capital subsidy is the same as in Proposition 1, with $\chi = \sigma n$ and therefore $\phi^{opt} = 1 - \alpha$.

R&D investment in the efficient allocation is

$$x = \frac{(1-a)}{a} \frac{n(1-\alpha)y}{(\eta+1)(r+n-1)},$$

whereas in the market economy, it is

$$x = \frac{\alpha}{(1-\tau)a} \frac{n(1-\alpha)y}{(\eta+1)(r+n-1)},$$

and therefore, the optimal subsidy is

$$\tau^{opt} = 1 - \frac{\alpha}{(1-a)} = 1 - \frac{\alpha(n+\chi)}{\chi},$$

where $a = \frac{n}{n+\chi}$.

As in Proposition 1, the remaining equations are the same for the efficient allocation and the decentralized economy, and thus both systems of equations produce the same steady-state solutions. ■

Appendix C: algorithm to compute dynamic welfare

This algorithm allows the computation of an approximation to the households' utility in stochastic environments when some of the endogenous variables are nonstationary. In a model with separable utility, the expected utility function of the representative household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(\mathbf{S}_t),$$

where $\mathbf{S}_t$ is the vector of endogenous variables that evolves according to a process $\mathbf{S}_t = g(\mathbf{S}_{t-1}, \boldsymbol{\omega}_t)$ and $\boldsymbol{\omega}_t$ is the vector of exogenous structural shocks.[23] The law of motion of $\mathbf{S}_t$ is obtained after solving the structural rational expectation model employing linear or non-linear methods. Assuming that $f(\boldsymbol{\omega}^t)$ is the probability distribution function of the shocks $\boldsymbol{\omega}^t = [\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_t]$ we may express the expected utility as

$$E_0 \sum_{t=0}^{\infty} \beta^t u(\mathbf{S}_t) = \int_{-\infty}^{\infty} \left(\sum_{t=0}^{\infty} \beta^t u(\mathbf{S}_t(\boldsymbol{\omega}^t)) \right) f(\omega^t) d\boldsymbol{\omega}^t \approx \frac{1}{M} \sum_{i=1}^{M} \left[\sum_{t=0}^{\infty} \beta^t u(\mathbf{S}_t(\boldsymbol{\omega}_i^t)) \right];$$

i.e., the unconditional mean can be approximated employing a standard Monte Carlo method. The accuracy of the algorithm is proportional to $\frac{1}{\sqrt{M}}$.

In our particular case, we solve the model by linear methods and truncate the maximum length $T = 500$ years of shocks.[24] We simulate the endogenous stationary variables $c_t(\boldsymbol{\omega}_i^t)$, $g_t(\boldsymbol{\omega}_i^t)$ and $l_t(\boldsymbol{\omega}_i^t)$. We reconstruct the nonstationary variables $A_t^{\max}$ as $A_t^{\max}(\boldsymbol{\omega}_i^t) = A_{t-1}^{\max}(\boldsymbol{\omega}_i^{t-1}) g_t(\boldsymbol{\omega}_i^t)$ with $A_0^{\max} = 1$ and $C_t(\boldsymbol{\omega}_i^t) = c_t(\boldsymbol{\omega}_i^t) A_t^{\max}(\boldsymbol{\omega}_i^t)$. We then compute the truncated cumulative discounted utility

$$U_i = \sum_{t=0}^{T} \beta^t \left[\log\left(C_t(\boldsymbol{\omega}_i^t)\right) - \mu_t(\boldsymbol{\omega}_i^t) \frac{l_t(\boldsymbol{\omega}_i^t)^{1+\psi} - 1}{1+\psi} \right].$$

Finally, we compute the unconditional expected utility $\frac{1}{M} \sum_{i=0}^{M} U_i$ over $M = 10,000$ simulations.

Appendix D: data sources

GDP data for the period 1950–2007 are taken from the *Bureau of Economic Analysis*. Data on civilian noninstitutional population aged 16 and older for the period 1950–2007 are taken from the *Bureau of Labor Statistics*. Data on civilian unemployment rates and gross private domestic investment for the period 1950–2007 and average weekly hours worked in all

[23] Typically, utility depends only on a subset of endogenous variables, such as consumption and labor.

[24] We use Dynare to solve and simulate the model.

private industries for the period 1964–2007 are taken from *St. Louis Fed* FRED database. Information about business turnover for the period 1990–2003 are provided by the *U.S. Small Business Administration*. Evidence for survival rates in the 1963 and 1976 cohorts is obtained from Dunne et al. (1988) and Audretsch (1991), respectively. Data on R&D expenditures for the period 1953–2007 are taken from the *National Science Foundation*.

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