Navigating by Falling Stars: Monetary Policy with Fiscally-driven Natural Rates *

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We study a new type of monetary-fiscal interaction in a heterogeneous-agent New Keynesian model with a fiscal block. Due to household heterogeneity, the stock of public debt affects the natural interest rate, forcing the central bank to adapt its monetary policy rule to the fiscal stance to guarantee that inflation remains on its target. There is, however, a minimum level of debt below which the steady-state inflation deviates from its target as the ZLB binds. We analyze the response to a debt-financed fiscal expansion and quantify the impact of different timings in the adaptation of the monetary policy rule, as well as the performance of alternative monetary policy rules that do not require an assessment of the natural rates. We validate our findings with a series of empirical estimates.

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1 Introduction

After the demise of the Bretton Woods system in 1971, a consensus emerged among academics and policymakers in which central banks were entrusted with the task of controlling inflation, while treasuries had to guarantee debt sustainability. This separation of tasks was enshrined in the Maastricht Treaty in 1992. The European Union member states gave birth to the European Central Bank with the primary objective of preserving price stability. At the same time, national treasuries were subject to a set of fiscal rules aimed at preventing debt and deficits from increasing above certain thresholds.

Economics provided intellectual support to this arrangement. In the basic representative-agent New Keynesian model or RANK (Woodford, 2003; Galí, 2008), the central bank can always deliver on its price stability mandate if certain conditions are satisfied. First, the treasury prevents debt from exploding. Second, the central bank responds forcefully enough to changes in inflation; i.e., it follows the so-called Taylor principle (or a variation of it). Third, the central bank tracks the natural rate r^* in the long run, and the latter is high enough to prevent the zero lower bound (ZLB) on nominal interest rates from becoming a binding constraint.[1]

The key result in the RANK framework is that the natural rate only depends on structural parameters, such as the household discount factor or productivity growth, which many economists felt confident to assume as remaining constant or evolving slowly according to secular trends.

Interestingly, this key result breaks down once we deviate from the complete-market representative-agent framework. Instead, consider a heterogeneous-agent New Keynesian (HANK) model, as popularized by Kaplan et al. (2018) and Auclert et al. (2018), among many others. These models incorporate a continuum of atomistic households subject to idiosyncratic risk who can save only using a limited set of non-state-contingent instruments. One important feature of HANK models is the fact that the natural rate depends on the stock of public debt, as originally pointed out by Aiyagari and McGrattan (1998), and more recently by Rachel and Summers (2019). The intuition is simple: given market incompleteness, the stock of public debt determines how much households can self-insure against negative idiosyncratic shocks and, therefore, the interest rate at which the savings market clears.

[1]In this paper, the natural rate is the real interest rate in the deterministic steady state of the economy. This concept is different (although often confused) from the neutral interest rate, which is the real interest rate in a counterfactual economy with flexible prices. The natural and neutral interest rates can be different if, for instance, nominal price rigidities make the deterministic steady state of the economy diverge from the counterfactual deterministic steady state with flexible prices. See Platzer et al. (2022) for a careful comparison of the natural vs. the neutral interest rate. The authors also call the natural rate the long-run r^* and the neutral rate the short-run r^*, but we find that terminology less useful since, in both cases, we are referring to the interest rate for a one-period bond.

The link between debt and natural rates opens the door to a form of monetary-fiscal interaction that has been, so far, unexplored. If the treasury changes the long-run stock of public debt, this decision necessarily moves the natural rate. The central bank should then either incorporate the new natural rate into its Taylor rule or ignore it, which will bias long-term inflation expectations.

Our main argument goes beyond HANK models. Overlapping generation (OLG) models, for instance, also share that feature, as in Eggertsson et al. (2019) and Aguiar et al. (2023). The latter authors introduce a tractable New Keynesian OLG model to analyze the global equilibrium dynamics of inflation, interest rates, and labor earnings in response to changes in the stock of public debt. Furthermore, the empirical literature has documented that this channel is significant in the data. See, for example, Rachel and Summers (2019).

Fiscally-driven natural rates vs. the FTPL. The reader should note that the interaction we focus on is conceptually different from the one analyzed under the fiscal theory of the price level (FTPL). The core insight of the FTPL, dating back to Sargent and Wallace (1981), is that if the treasury is not committed to guaranteeing debt sustainability, the central bank may be forced to accommodate fiscal expansions to prevent debt from exploding. In these circumstances, inflation is determined by the need for stabilizing public debt. This is the kind of logic behind the Maastricht Treaty discussed above. In our argument, instead, the treasury collects taxes, spends, and funds deficits by issuing debt according to a fiscal rule that stabilizes the long-run real debt level. Fiscal policy affects the conduct of monetary policy not by refusing to stabilize long-term debt but by shifting natural rates.

Our analysis. We postulate a HANK model with a central bank under inflation targeting that sets nominal interest rates following a standard Taylor rule and a treasury. The economy has a unique steady state in which the debt target of the treasury pins down the natural rate: the higher this target, the higher the natural rate is. Steady-state inflation deviates from the central bank's target in proportion to the difference between the natural rate and the intercept of the Taylor rule. Thus, to ensure price stability, the central bank should change the Taylor rule intercept depending on the treasury's long-run debt target.

In comparison, the RANK version of the model (when we shut down household heterogeneity) has a natural rate that only depends on household preferences (as in the conventional New Keynesian framework), and the central bank can ignore the treasury's long-run debt target.

There is nevertheless a situation in which the central bank in a HANK world cannot deliver on its long-run mandate even if it is willing to adapt its rule to the fiscal position. If the debt target is low enough, the natural interest rate becomes negative. If the natural rate is so negative that its sum with the inflation target is below zero, then the ZLB will be binding in the long term, and inflation will be equal to the opposite of the natural rate. Thus, there is a *minimum debt level compatible with the inflation target*. For any debt target below that minimum level, the central bank fails to deliver on its mandate.

Among many possible experiments, we focus on the response of the economy to a debt-financed fiscal expansion because we find it illustrates our argument better than other exercises. At time zero, the treasury announces an increase in its debt target, which temporarily expands the room for government spending. The economy then converges to a new steady state characterized by both higher debt and natural rate. To achieve its inflation target in the long run, the central bank must increase the intercept in its Taylor rule, which raises the steady-state level of real and nominal rates. If it fails to do so, inflation increases both in the long and in the short term, though the impact on real variables is negligible. Another way to think about this result is that, by not adjusting the intercept in its Taylor rule, the central bank is implicitly changing its inflation target, which does not have much of a long-run effect on real variables.

The increase in inflation after the fiscal expansion is larger than in the counterfactual RANK model. The different inflation dynamics between HANK and RANK are mainly due to the different paths for employment and consumption. In the RANK model, consumption declines entirely due to the intertemporal substitution, as households save more to profit from the increase in real interest rate increase. This channel is also present in the HANK model, especially for wealthy households. However, it is partially compensated by an income effect that pushes consumption up, as poor low-income households can now increase their consumption due to better self-insurance allowed by the existence of more bonds.

We explore a series of extensions of our model. Our first extension is to analyze the impact of timing decisions regarding the change in the policy rule. We modify the previous experiment and consider that the fiscal expansion is announced 12 quarters in advance. We then study the impact of three different scenarios. First, changing the monetary policy rule on the announcement date. Second, changing the rule on the date of the actual change. Third, changing it 12 quarters later than the actual change. While the three scenarios yield similar dynamics for inflation after the change in the rule, the later the announcement, the higher the inflation *prior* to the change is.

Our second extension is to explore a proposal by Orphanides and Williams (2002), who argue that a *differential* rule in which deviations of inflation from its target affect the change in nominal interest rates avoids the need for an assessment of the natural rate. We show how such a rule produces much less volatility in inflation while the dynamics of real variables are relatively similar. This suggests that such a rule may be a good candidate to deal with the endogeneity of natural rates to permanent fiscal policy shocks.

Our third extension is to incorporate long-term public debt into the model. The muted impact on real variables in the short run of sticking to the old monetary policy rule after a debt-financed fiscal change is an artifact of considering short-term debt. If we extend the model to include long-term public bonds, the impact becomes significant in the short run. This is a consequence of the Fisher effect, as the surprise increase in inflation reduces the real value of debt. This reduction leads to a redistribution of real resources from wealthy households towards the treasury, which in turn allows it to engage in a more aggressive fiscal expansion.[2]

In Section 7, we present empirical evidence validating the mechanisms in our model. First, we show how a standard estimate of the natural rate (Lubik and Matthes, 2015) increases in response to a rise in the debt-to-GDP ratio in a magnitude similar to the one predicted by the model. Second, we develop an analytical expression linking deviations of long-term inflation from the central bank inflation target to the *policy gap* between the natural rate and the central bank's long-term rate implicit in its reaction function. We evaluate this expression using market data on long-term interest rates and inflation expectations and find significant policy gaps, especially in the post-pandemic period.

Literature review. This paper is related to the large literature on monetary-fiscal interactions, particularly under the FTPL (see Leeper, 1991; Sims, 1994; Cochrane, 1999; Woodford, 1995; or Schmitt-Grohe and Uribe, 2000; among others). More recently, Bianchi et al. (2022) analyze a model in which a monetary-led and a fiscally-led policy mix coexist at the same time, as the central bank accommodates unfunded fiscal shocks causing persistent movements in inflation. Bigio et al. (2023) study the expectation of a monetary-fiscal reform. Under the reform, monetary policy is temporarily obliged to provoke inflation to aid the treasury in making its debt sustainable. After the reform, debt and inflation stabilized again.

This paper also contributes to the literature analyzing fiscal and monetary policies in HANK models, including Oh and Reis (2012), Kaplan et al. (2018), Auclert et al. (2018), Hagedorn et al. (2019), McKay and Reis (2021), Wolf (2021), or Ferriere and Navarro (2018). Bayer et al. (2023) analyze how a permanent increase in the ratio of public debt to GDP increases real public bond yield in the long run. Hagedorn (2016) shows that prices and inflation are jointly and uniquely determined by fiscal and monetary policy in a HANK model with nominal debt. Kaplan et al. (2023) analyze the FTPL in the context of a heterogeneous-agent model with flexible prices. In their model, a permanently higher deficit is associated with a lower steady-state real interest rate and less real public debt, as well as a higher long-run inflation rate for a given setting of monetary policy.

[2]The empirical relevance of the Fisher effect has been documented by Doepke and Schneider (2006a), Adam and Zhu (2016), or Ferreira et al. (2023), among others.

This paper is also related to the literature on the estimation of the natural rate and its implications for monetary policy. There are different methods to estimate the natural rates, including semi-parametric methods (such as Laubach and Williams, 2003 or Holston et al., 2017), nonstructural time series methods (Lubik and Matthes, 2015), or methods based on extracting information on the expected long-run real interest rate from bond prices (e.g., Christensen and Rudebusch, 2019 or Davis et al., 2023). Since these methods do not always yield similar results, there is a significant degree of uncertainty in the estimation of the natural rate. This may lead to misperceptions about the level of natural rates, with implications for the monetary policy stance, as discussed by Ajello et al. (2020). Chortareas et al. (2023) estimate a time-varying Taylor rule for the U.S. and document how the Federal Reserve has occasionally misread the natural rate of interest. Finally, Bauer and Rudebusch (2020) show how accounting for time variation in natural rates is crucial for understanding the dynamics of the yield curve, a key object in monetary policy transmission.

The rest of the paper is structured as follows. Section 2 introduces our HANK model and Section 3 its calibration and computation. Section 4 explores the monetary-fiscal interactions in our model in the long run, while Section 5 analyzes the dynamics of such interaction. Section 6 discusses several extensions of our model. Section 7 presents empirical evidence validating the main elements of our analysis, and Section 8 concludes.

2 A heterogeneous-agent model with monetary and fiscal policy

To investigate monetary policy with fiscally-driven natural interest rates, we introduce a baseline discrete-time HANK model with monetary and fiscal policy. We follow Auclert et al. (2023) and assume that wages are subject to nominal rigidities and hours are determined by a union on behalf of the workers.[3] Firms produce the final good with the labor supplied by the union. The model is closed by a monetary policy authority, which determines the nominal interest rate, and a treasury, which taxes, spends, and issues public debt.

Households. There is a continuum of households indexed by $i \in [0, 1]$. Households derive utility from consumption, $c_{i,t}$, and disutility from working $n_{i,t}$ hours. They can only save in a nominal public bond. Given a discount factor β, the intertemporal problem solved by each

[3] Auclert et al. (2023) show that HANK models with wage rigidities and flexible prices can simultaneously match plausible estimates of marginal propensities to consume (MPCs), marginal propensities to earn (MPEs), and fiscal multipliers. In contrast, HANK models with price rigidities and frictionless labor markets fail at doing so. The drawback of sticky prices is that they induce countercyclical mark-ups. Unless we distribute the corresponding profits in an *ad-hoc* way to ensure the right allocation of income, the MPCs and MPEs of the model cannot match the data.

household is:

$$V(a_{i,t}, z_{i,t}) = \max_{c_{i,t}, a_{i,t+1}} u(c_{i,t}) - v(n_{i,t}) + \beta \mathbb{E}_t[V(a_{i,t+1}, z_{i,t+1})]$$
$$\text{s.t. } c_{i,t} + a_{i,t+1} = (1 + r_t)a_{i,t} + (1 - \tau)\frac{W_t}{P_t} z_{i,t} n_{i,t},$$
$$a_{i,t+1} \geq 0,$$

where $a_{i,t}$ is the household's asset position in real terms at the start of the period, $z_{i,t}$ is the idiosyncratic labor productivity, r_t denotes the *ex-post* real return of bonds in period t, W_t is the nominal wage, and P_t is the price level. Labor income is taxed at a constant rate τ. Households cannot short bonds, i.e., $a_{i,t+1} \geq 0$.

At time t, household i works $n_{i,t}$ hours. A union chooses these hours on behalf of households. Each hour provides $z_{i,t}$ units of effective labor, so that aggregate hours are $N_t = \int_0^1 z_{i,t} n_{i,t} di$. The idiosyncratic shock $z_{i,t}$ follows a first-order Markov chain with mean $\mathbb{E}_t z_{i,t+1} = 1$. Agents take their hours $n_{i,t}$ as given. We assume a proportional allocation rule for labor hours, with $n_{i,t} = N_t$. The nominal wage W_t is determined by union bargaining as specified below.

Unions. We follow a standard formulation for sticky wages with heterogeneous agents, similar to Auclert et al. (2018) and Auclert et al. (2021b). Suffice it to say that the union aggregates different labor tasks provided by the households into a homogeneous labor service. Appendix A describes the details. The union employs all households for the same number of hours N_t and sets nominal wages by maximizing the welfare of the average households subject to a penalty term on the fluctuation of nominal wages over the central bank's inflation target $\overline{\pi}$.

Solving the problem leads to a wage Phillips curve:

$$\log\left(\frac{1+\pi_t^w}{1+\overline{\pi}}\right) = \kappa_w \left[-\frac{\epsilon_w - 1}{\epsilon_w}(1-\tau)\frac{W_t}{P_t}\int u'(c_{it}) z_{it} di + v'(N_t)\right] N_t + \beta \log\left(\frac{1+\pi_{t+1}^w}{1+\overline{\pi}}\right), \tag{1}$$

where ϵ_w is the elasticity of substitution between different labor tasks, κ_w is the slope of the Phillips curve (itself a nonlinear function of other parameters of the model), and $\pi_{wt} \equiv \frac{W_t}{W_{t-1}} - 1$, is the nominal wage inflation rate.

Firms. There is a continuum of identical firms. Firms produce final goods using a constant return-to-scale technology $Y_t = \Theta N_t$, where N_t is aggregate labor and $\Theta > 0$ is a constant productivity parameter. The real wage is given by $\frac{W_t}{P_t} = \Theta$.

From these equations, wage inflation is equal to goods inflation, $\pi_t = \pi_{wt}$, where the latter is defined as $\pi_t \equiv \frac{P_t}{P_{t-1}} - 1$.

Monetary policy. The central bank sets the nominal interest rate on nominal bonds i_t according to a standard monetary policy rule that responds to inflation, and it is subject to a ZLB (below, we will explore some of the consequences of this bound being binding):

$$\log(1+i_t) = \max\left\{\log(1+\overline{r}) + \log(1+\overline{\pi}) + \phi_\pi \log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right), 0\right\}, \tag{2}$$

where $\phi_\pi \geq 1$ is the slope of the Taylor rule, $\overline{\pi}$ is the inflation target, and $\overline{r}$ is the real rate intercept.

Fiscal Policy. The treasury (which we interpret as the government in our economy) collects labor taxes from households and spends on government consumption goods. The treasury can issue one-period nominal bonds. Tax collection is given by:

$$T_t = \int_0^1 \tau \frac{W_t}{P_t} z_{i,t} n_{i,t} di.$$

Later, in Section 6, we will introduce long-term debt.

Public debt B_t accumulates according to:

$$P_t B_t = (1+i_{t-1}) P_{t-1} B_{t-1} + P_t(G_t - T_t),$$

where B_t denotes the stock of bonds in real terms and G_t and T_t are real government consumption and real taxes, respectively. If we define *ex-post* real rates as $(1+r_t) \equiv (1+i_{t-1})\frac{P_{t-1}}{P_t}$, we can express the government budget constraint as $B_t = (1+r_t)B_{t-1} + G_t - T_t$.

Tax collection T_t follows an endogenous process determined by the evolution of its underlying component variables. In comparison, government consumption, G_t, follows a fiscal rule depending on the expenditure $\overline{G}$ and the debt target $\overline{B}$:

$$G_t = \overline{G} - \phi_G(B_{t-1} - \overline{B}), \tag{3}$$

where $0 < \phi_G < 1$ controls the speed of fiscal adjustment when debt is not at its target. This fiscal rule is studied by Auclert et al. (2020, Section 5.3). It is also closely related to the fiscal rules studied by Auclert and Rognlie (2018). Kaplan et al. (2023), on the other hand, consider a rule in which the fiscal deficit adjusts instead of government consumption. Below, we will explore the consequences of unexpected shocks to $\overline{B}$. Government consumption does not enter into the utility of the households (or, equivalently, it enters in a separable way from private consumption and labor supply).

Aggregation and market clearing. In equilibrium, the labor, bond, and good markets clear:

$$N_t = \int_0^1 z_{i,t} n_{i,t} di,$$
$$B_t = \int_0^1 a_{i,t+1} di,$$
$$C_t = \int_0^1 c_{i,t} di,$$

and the aggregate resource constraint holds: $G_t + C_t = Y_t$.

3 Calibration and computation

We calibrate our model at quarterly frequency by borrowing standard parameter values from the literature and matching observations of the U.S. economy. Table 1 summarizes the calibration, which we explain below in some more detail.

Preferences. We assume log utility over consumption: $u(c) = \log(c)$. The disutility over hours is parameterized using a function with a constant Frisch elasticity: $v(n) = \nu_\varphi n^{1+\frac{1}{\varphi}}/(1+\frac{1}{\varphi})$. We set the Frisch elasticity φ to 0.5. The preference shifter ν_φ is calibrated so that, given all other parameters, total employment is 1 in the steady state. This is an immaterial normalization. In our baseline calibration, this normalization implies $\nu_\varphi = 0.881$. We calibrate the discount factor to match a real interest rate of 1% annually. This implies a quarterly discount factor of $\beta = 0.991$ and an annual discount factor of 0.965.

Income process and borrowing limit. The persistence and standard deviation of income shocks are taken from estimates by Floden and Lindé (2001) of the U.S. wage process. We set the persistence of the income process to match a persistence of 0.91 yearly and the standard deviation of innovations to match the standard deviation of log gross earnings of 0.92. We first convert these values to quarterly frequency and then approximate the income process with a Markov chain with 11 discrete states calculated using the method by Rouwenhorst (1995).[4]

We discretize the asset space using a double-exponential transformation of a uniformly spaced grid using 500 grid points, with a minimum asset level of $\underline{a} = 0$ (the borrowing limit) and a maximum asset level of $\overline{a} = 150$. We solve the household problem using the endogenous grid method (Carroll, 2006; Barillas and Fernández-Villaverde, 2007).

Production. We normalize the steady state quarterly output and total factor productivity

[4]The quarterly persistence is calculated as $\rho_Q = 0.92^{1/4} \approx 0.977$ and the quarterly standard deviation of innovations as $\sqrt{(0.91^2/(\sum_{t=0}^{3} \rho_Q^{2t})} \approx 0.476$.

Table 1: Calibration parameters, Baseline model

	Parameter	Value	Target/Sources
	Preferences		
σ	Elasticity of intertemporal substitution	1	Standard
φ	Frisch elasticity of labor supply	0.5	Standard
ν_φ	Disutility of labor parameter	0.881	$N_{ss} = 1$
β	Quarterly discount factor	0.991	1% real interest rate in DSS
	Income process		
ρ_e	Persistence income process (annual)	0.91	Floden and Lindé (2001)
σ_e	Std. dev. idiosyncratic shock (annual)	0.92	Floden and Lindé (2001)
	Production		
Y	Quarterly output	1	Normalization
Θ	Constant level of TFP	1	Normalization
κ_w	Slope of the wage Phillips curve	0.1	Aggarwal et al. (2023)
ϵ_w	Elasticity of substitution	10	Standard
	Fiscal policy		
r	Real interest rate (annual)	0.01	Baseline case
$\overline{B}$	Debt target	2.8	Debt-to-GDP 70%
$\overline{G}$	Government spending target	0.2	Spending-to-GDP 20%
τ	Tax rate	0.207	B constant in DSS
ϕ_G	Coefficient in the fiscal rule	0.1	Baseline case
	Monetary policy		
ϕ_π	Taylor rule coefficient	1.25	Standard
$\overline{\pi}$	Inflation target (annual)	0.02	Standard

Θ to one. This implies that total hours equal output in the steady state. We set the value of elasticity between labor tasks ϵ_w to 10, which is a standard value in the literature (e.g., Wolf, 2021). We take the slope of the wage Phillips curve κ_w from Aggarwal et al. (2023) and set it to 0.1.

Fiscal policy. We assume a conventional target of government consumption of 20% of GDP, which is close to U.S. data and is also the number used by Auclert et al. (2018). Because we have normalized quarterly output to one, this implies that $\overline{G} = 0.2$.

In our simulations, we consider different levels of public debt in the steady state. We take as a benchmark the case in which the annual real interest rate is 1%, and public debt stands at 70% of annual GDP. In our quarterly model, the stock of public debt at which the real interest rate is at its steady state value is, therefore, $\overline{B} = 4 \times 0.7 = 2.8$. The value of the tax rate is chosen to balance the budget in the steady state so that public debt remains constant, i.e., total tax revenue exactly covers government spending plus interest payments on public debt $T_{ss} = G_{ss} + r_{ss} \times B_{ss} = 0.2 + 0.01/4 \times 2.8 = 0.207$. Because $T_{ss} = \tau Y_{ss}$ and $Y_{ss} = 1$, this implies that $\tau = 0.207$. Finally, for the baseline case, we set the parameter ϕ_G, which governs how quickly government spending responds to deviations from the debt target, to 0.1.

Monetary policy. We parameterize the Taylor rule to achieve an inflation target of 2% annually and set the Taylor rule coefficient to 1.25.

A RANK version of the model. In order to gain further insight into our model, we also consider for comparison purposes a representative agent New Keynesian (RANK) version of our economy. The RANK model eliminates the idiosyncratic income shock by setting $z_{i,t} = 1$. All the other structural parameters are the same as in the baseline HANK model except that the value of the discount factor β is re-calibrated to 0.9975, such that the steady state of the RANK model coincides with that of the HANK.

Computation. Given the set of exercises we want to undertake with our HANK model (e.g., explore the effects of unanticipated long-term changes in fiscal policy), we follow the recent literature on heterogeneous agent models by solving nonlinearly for impulse responses to one-time, unanticipated aggregate shocks using the sequence-space method. More concretely, we rely on the computational toolkit developed by Auclert et al. (2021a) for both computation and the decomposition of impulse responses.

4 Monetary-fiscal interactions in the long run

We start by analyzing the deterministic steady state (DSS) of the model, where there are no aggregate shocks, but we still have idiosyncratic shocks at the level of households. First, we consider the demand for bonds. Second, we study the supply of bonds (determined by fiscal policy). Third, we find the pricing of nominal bonds (as pinned down by the monetary policy rule). We denote steady-state variables with the subindex "ss" except for real interest rates, for which we use the more standard r^*, as in our model this variable coincides with what is usually called the long-run natural rate. Also, since households still receive idiosyncratic shocks, we keep the t subindex for their variables.

Demand for bonds. The demand for bonds comes from households, who accumulate them to smooth consumption. By aggregating individual bond holdings at the DSS, we get:

$$A_{ss}(r^*) = \int_0^1 a_{i,t+1} di.$$

We show in Appendix B that the demand for bonds is a continuous and monotonically increasing function of the interest rate, with $\lim_{r^* \to \frac{1-\beta}{\beta}} A_{ss}(r^*) = \infty$ and $\lim_{r^* \to -\infty} A_{ss}(r^*) = 0$. The solid red line in Figure 1 displays the demand for bonds in our baseline calibration.

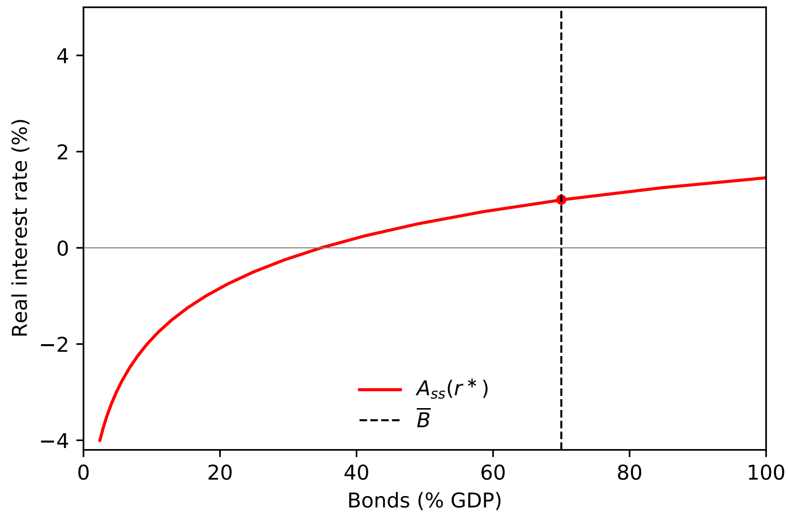


Figure 1: Determination of the steady-state real interest rate.

Supply of bonds. The supply of bonds comes from combining the budget constraint of the treasury, $B_{ss} = (1 + r^*)B_{ss} + G_{ss} - T_{ss}$, with the fiscal rule $G_{ss} = \overline{G} - \phi_G(B_{ss} - \overline{B})$. In Appendix C, we show how it has the form $B_{ss} = \overline{B}$. The supply of bonds is depicted as a vertical line in Figure 1.

Monetary policy rule. The DSS monetary policy rule (2) is approximated by $i_{ss} \approx \max\{\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}), 0\}$. This rule can be combined with the long-run Fisher equation $i_{ss} = r^* + \pi_{ss}$ to get:

$$r^* + \pi_{ss} \approx \max\{\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}), 0\}. \tag{4}$$

Notice that, in our model, the DSS long-run Fisher equation is just an identity.

It is well known that equation (4) has two solutions (Benhabib et al., 2002). First, we have the *non-binding ZLB* scenario, in which $\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}) > 0$, which gives a solution $r^* + \pi_{ss} \approx \overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi})$, or equivalently

$$\pi_{ss} \approx \overline{\pi} + \frac{r^* - \overline{r}}{\phi_\pi - 1}. \tag{5}$$

That is, steady-state inflation equals the central bank inflation target plus a term that accounts for the deviation between the intercept in the Taylor rule and the steady-state natural rate in the economy. Consequently, in order to guarantee that long-run inflation remains at its target, the central bank should equate the intercept in its Taylor rule to the natural rate. This is the standard prescription in the New Keynesian model.

Second, we have the *binding ZLB* scenario, in which $\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}) \leq 0$, and the maximum of the right-hand side is zero. In this case, the nominal rate is zero, and we get then that inflation equals minus the natural rate $\pi_{ss} = -r^*$.

The deterministic steady state. Now, we combine the three equations to characterize the DSS of the model. First, in Appendix C, we prove that, if $\phi_G > (1-\beta)/\beta$, then there exists a unique steady state.

Second, by equating supply and demand, we get $A_{ss}(r^*) = \overline{B}$. Given that the supply of bonds is vertical, the demand for bonds traces out the combinations of steady-state real interest rates that are compatible with the steady-state debt target. Moreover, because the demand for bonds is upward sloping, higher steady-state debt levels invariably lead to a higher steady-state real interest rate, and we can invert the function $A_{ss}(r^*)$ and get the relationship $r^*\left(\overline{B}\right)$.

Third, we combine this interest rate with the Taylor rule. When the ZLB does not bind, we assume that the central bank has purposely picked this intercept to guarantee that inflation remains on target. That is, we assume that $\overline{r} = r^*\left(\overline{B}\right)$ and hence $\pi_{ss}\left(\overline{B}\right) = \overline{\pi}$. This can be achieved if real interest rates are high enough. There is, however, a level of the debt target $\overline{B}^*$ defined as

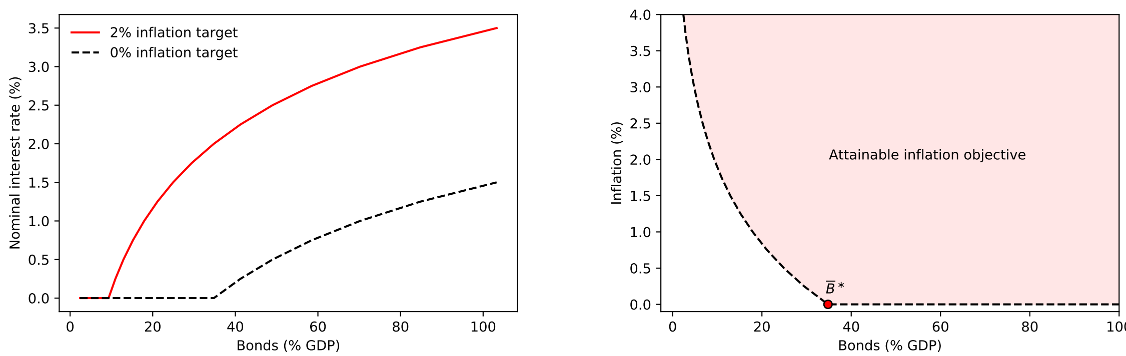
$$r^*\left(\overline{B}^*\right) + \overline{\pi} = 0,$$

such that it makes the nominal rate equal to zero. For debt targets below that level, $\overline{B} < \overline{B}^*$, the impossibility of nominal rates to be negative forces the central bank to tolerate steady-state inflation levels above its target, that is, the ZLB is binding, $i_{ss} = 0$, and inflation is pinned down by the condition $\pi_{ss} = -r^*\left(\overline{B}\right)$, for $\overline{B} < \overline{B}^*$. We denote the *minimum debt level compatible with the inflation target* $\overline{\pi}$ by $\overline{B}^*$. Only for debt targets above that level, $\overline{B} > \overline{B}^*$, can the central bank deliver on its inflation target in the long run.

We illustrate this result in Figure 2. Panel (a) displays the nominal interest rate for two different inflation targets, 2% (solid red) and 0% (dashed black). We see that for an inflation target of 2%, the minimum debt level $\overline{B}^*$ is 9% of GDP. In contrast, for 0%, it is 35%.[5] The dashed line in Panel (b) displays the frontier $\pi\left(\overline{B}^*\right)$ for positive inflation targets. The shaded area in Panel (b) shows the set of (non-negative) inflation targets that can be achieved in equilibrium for a varying level of debt. The level of debt B^* is the lowest level of debt compatible with an inflation objective of zero.[6]

[5]In these exercises, we let $\overline{G}$ adjust to ensure the treasury satisfies its budget constraint.

[6]Our fiscal rule delivers different values for the primary surplus $T_{ss} - G_{ss}$ as a function of the debt target (see Figure 13 in Appendix D). As discussed in Kaplan et al. (2023), our fiscal rule avoids the problems of multiplicity of steady states even with persistent deficits.



(a) Steady-state nominal interest rate

(b) Steady-state inflation

Figure 2: Steady-state nominal interest rate and inflation for different inflation targets

Note: The graphs are based on the baseline calibration but allow the level of bonds $\overline{B}$ to vary. In Panel (a), the dashed line shows results for an inflation target of 0% and the red solid line for 2%. The area in Panel (b) shows the combinations of (non-negative) inflation objectives that can be achieved in equilibrium for a varying level of debt. The level of debt B^* is the lowest level of debt compatible with an inflation objective of zero.

More assets. At first sight, the results in the previous paragraphs may seem to depend on the absence of other assets, like private productive capital, in which the households can save (see, for a model with public debt and capital, Bayer et al., 2023). However, this is not the case: our results would still hold but at the cost of less transparency in the intuition. Imagine, for instance, a world where we add to our model productive physical capital with a standard Cobb-Douglas production function. The net return on capital is equal to its marginal productivity minus depreciation. If marginal productivity is sufficiently low (i.e., there is much capital accumulated because there is not enough public debt for the households to save on) with respect to depreciation, the net return on capital is negative (see for a quantitative model that delivers negative net returns on capital, Barro et al., 2022). By a non-arbitrage condition, the net return on capital must be equal to the real return on bonds. If, in addition, the inflation target is not high enough, the ZLB is binding, and the central bank cannot deliver price stability. Hence, while the presence of other assets might change the quantitative shape of Figure 2, the main message of this figure carries on: the central bank needs enough outstanding public debt to fulfill its role.

Optimal monetary policy. So far, we have just focused on monetary policy rules. One natural question is how our results generalize to the case of optimal monetary policy, either under discretion or commitment. While a complete analysis of the optimal monetary policy response in the context of this HANK model goes beyond the aim of this paper, we provide some initial insights.[7] Let denote by π^*_{ss} the inflation rate in the DSS of the optimal problem,

[7] Optimal monetary policy in HANK economies requires dealing with infinite or high dimensional problems,

be that under commitment or discretion. The long-run Fisher equation $i_{ss} = r^* + \pi^*_{ss}$ still holds in this case. That implies that, given this optimal inflation level, (i) any change in the long-run stock of debt will imply a change in the long-run nominal rate, and (ii) there is a minimum debt level compatible with the optimal inflation target. Our analysis highlights that optimal monetary policy must consider these two conditions, i.e., the design of monetary policy cannot be separated from fiscal policy.

5 A surprise debt-financed fiscal expansion

Next, we analyze the dynamic impact of a debt-financed fiscal expansion, that is, a permanent expansion in debt, both in the HANK and RANK versions of our model. As we mentioned in the introduction, we consider that this exercise is the best way to illustrate the mechanisms we want to highlight in this paper.

More concretely, we consider that the economy starts at the DSS described in Section 3 when the treasury announces a previously unexpected increase in its debt target $\overline{B}$. In particular, $\overline{B}$ increases permanently on impact by 10 p.p. in terms of the initial GDP.

To ensure that the treasury satisfies its intertemporal budget constraint and that public debt remains constant in the new DSS, this higher $\overline{B}$ must be financed either with higher taxes τ or lower spending (or a combination of both). A simple assumption, which helps to clarify the forces at work in our model, is to assume that the treasury reduces $\overline{G}$, the intercept of the fiscal rule (3) on impact. Thus, the primary surplus exactly offsets the increase in interest payments on public debt.

In the HANK model, the required change in the intercept is $\Delta\overline{G} \approx \Delta T_{ss} - (\Delta r^* \overline{B}_{ss} + r^* \Delta \overline{B}_{ss}) = -0.02 - (0.18\% \times 70 + 1\% \times 10) \approx -0.27$ (we will explain momentarily the reason why we have a $\Delta r^* > 0$). A similar calculation for the RANK version of the model implies that the intercept of the fiscal rule in that model must decrease by only 0.11.

5.1 Comparison of steady states

With the change in $\overline{B}$ and the associated move in $\overline{G}$, we can compute the new DSS and the transition to it. First, we compare the old and new steady states in Table 2.

The key finding of Table 2 is that the r^* increases by 18 b.p. in the new DSS of the HANK model. This value is close to the 25 b.p. computed by Bayer et al. (2023) in response to a very persistent increase in debt of 10 p.p. The intuition is simple: a higher supply of bonds lowers their price, i.e., it raises their yield. In terms of Figure 1, the vertical black line moves

as discussed in Nuño and Thomas (2022), Bhandari et al. (2021), or Dávila and Schaab (2022).

	Initial steady state	New steady state		Difference	
		HANK	RANK	HANK	RANK
Bonds (% GDP)	70.00	80.00	80.00	10.00	10.00
Real interest rate	1.00	1.18	1.00	0.18	0.00
Nominal interest rate	3.02	3.20	3.02	0.18	0.00
Output	100.00	99.89	99.96	-0.11	-0.04
Consumption	80.00	80.15	80.07	0.15	0.07
Govt. consumption	20.00	19.73	19.89	-0.27	-0.11
Tax revenue	20.70	20.67	20.69	-0.02	-0.01
Primary surplus (% GDP)	0.70	0.94	0.80	0.24	0.10

Table 2: Steady state in the baseline HANK model and the RANK model
Note: The nominal interest rate in the initial DSS is 3.02% and not 3.00% because it satisfies the non-linear version of the Fisher equation $i_{ss} = 1.01 \times 1.02 - 1 \approx 3.02\%$.

to the right, and it crosses the demand curve (red line) at a higher interest rate.

Therefore, if the central bank wants to preserve price stability, it must raise the intercept in its monetary policy rule $\overline{r}$ by the same amount. This is why when we computed above $\Delta\overline{G}$, we had the term $\Delta r^* > 0$. In other words, not only must $\overline{G}$ fall to allow for the service of the extra debt, but it must fall as well to accommodate the higher $r^* > 0$, for a total reduction of government spending of 27 b.p.

The other side of the coin of the adjustments in monetary and fiscal policy after an increase in $\overline{B}$ is that households can consume 15 b.p. more due to the positive wealth effect of lower government spending (recall that the household receives the higher debt services). The higher consumption leads households to work less, thus reducing output by 11 b.p. (the labor supply effect of government spending changes through wealth effects is well-known since at least Christiano and Eichenbaum, 1992).

Table 2 also reports the results for the RANK version of the model (recall that the initial DSS is the same for both versions of the model; only the new steady states are different). The key difference between the RANK and the HANK is that the steady-state natural rate r^* does not change in the RANK case, while it does in HANK. This is because the long-run demand for bonds in the RANK model is perfectly elastic, with steady-state real interest rates pinned down by the household's impatience, $r^*_{RANK} = \frac{1}{\beta}$. Thus, the central bank does *not* need to adjust its Taylor rule to the new fiscal situation.

Nonetheless, the fiscal rule has to adjust, though to a lesser extent than in the HANK version. The reason can be found in the debt sustainability equation $-r^* B_{ss} = G_{ss} - T_{ss}$. In the RANK case, the change in government spending needs to match the increase in debt and the decline in taxation due to the lower output $\Delta G^{RANK}_{ss} = -r^* \Delta B_{ss} - \left(-\Delta T^{RANK}_{ss}\right)$.

5.2 Dynamics after the announcement

Next, we study the dynamic response of the economy after the increase in the debt target $\overline{B}$ introduced above. The solid red lines in Figure 3 display the transitional dynamics in the baseline HANK economy.

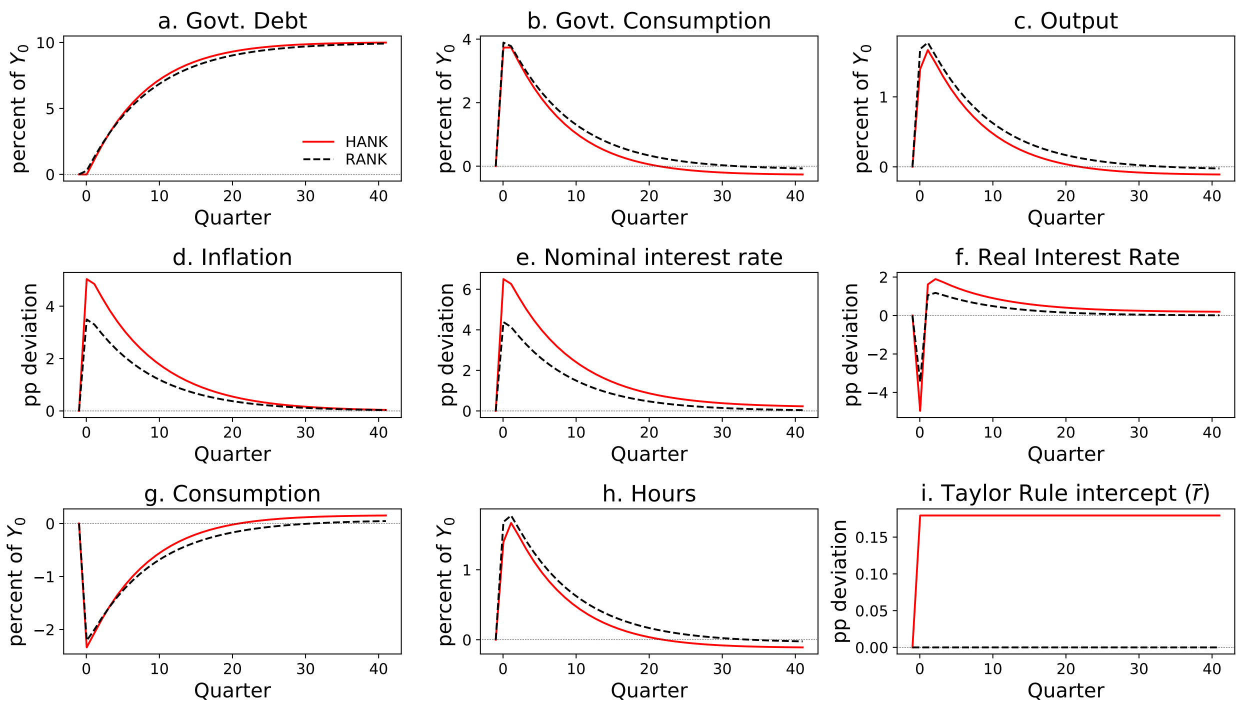


Figure 3: Dynamics after a surprise debt-financed fiscal expansion

The treasury converges to a higher $\overline{B}$ (Panel b) by temporarily spending more for around 20 quarters (Panel b). Afterward, government spending is permanently lower, as it needs to service a higher debt, and output is smaller. The transitory increase in government consumption leads to an expansion in output (Panel c) and inflation (Panel d) through the standard New Keynesian transmission of demand shocks. Annualized inflation peaks at 6.8%, 4.8 p.p. above the 2 % target, and GDP increases by 1.7 p.p.

To stabilize prices, the central bank increases nominal rates by around 6.2 p.p., which is higher than inflation (Panel e). This increase is mainly due to the term $\phi_\pi \log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right)$ in the Taylor rule, which accounts for roughly 6 p.p., while the intercept increases by 18 b.p. (Panel i). While this increase in the intercept plays a minor role in the initial periods, its importance increases as the short-term dynamics fade out. Ex-post real rates increase by 1.4 p.p. (Panel f). One interesting feature of this experiment is the initial decline in the real interest rate. On impact, the surprise increase in inflation reduces ex-post real interest rates, as they are defined as $r_t \equiv \frac{1+i_{t-1}}{1+\pi_t} - 1$. The increase in real rates induces households to consume less

(Panel g) and work more (Panel h). Thus, households save more (and absorb the new public debt). Because of the increase in labor supply (and, with it, of output), the fall in private consumption is less than the increase in government consumption.

As time progresses, these short-run dynamics fade out, and the economy converges to the new DSS with higher debt, lower government spending, higher private consumption, and higher real interest rates.

The dashed black lines in Figure 3 show the response of the RANK version of the model. The transmission of debt-financed fiscal expansion is now more muted. Inflation increases by 3.5% on impact in the RANK model, compared to 5% in the HANK (see Panel d in Figure 3).

To understand better the difference between the two models, we express the wage Phillips curve (1) as an infinite discounted sum:

$$\log\left(\frac{1+\pi_0}{1+\overline{\pi}}\right) = \sum_{t=0}^{\infty} \beta^t \kappa_w \left[-\frac{(\epsilon_w - 1)}{\epsilon_w}(1-\tau)\int u'(c_{i,t}) z_{it} di + v'(N_t)\right] N_t,$$

that is, inflation on impact is a function of the entire future evolution of the cross-sectional average of marginal utilities $\int u'(c_{i,t}) z_{it} di$, labor disutility $v'(N_t)$, and hours worked N_t.

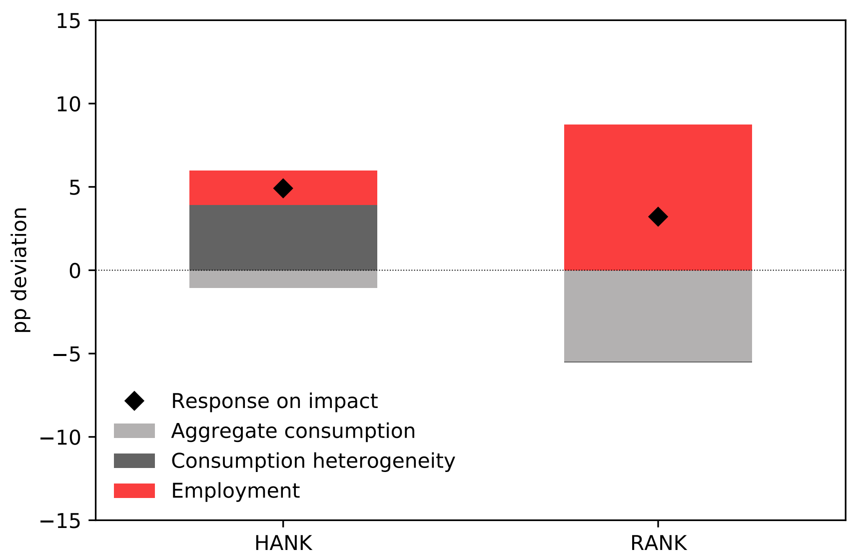


Figure 4: Decomposition of the response of inflation on impact

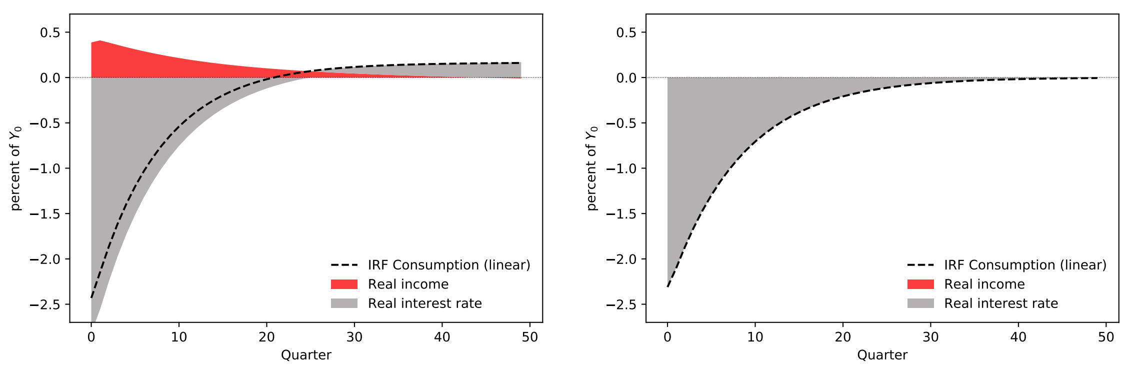
Note: The decomposition uses the Jacobians of the wage New Keynesian Phillips Curve and then combines them with the general equilibrium response of aggregate consumption, a consumption heterogeneity measure, and employment in each model.

Figure 4 decomposes the impact on inflation in terms of aggregate consumption, consumption heterogeneity, and employment.[8] The higher increase in inflation in HANK is the

[8]We employ the Jacobians of the wage Phillips curve and combine them with the general equilibrium

combination of three factors. First, the stronger rebound in aggregate consumption (Panel g in Figure 3) reduces the deflationary effect of the higher marginal utility of aggregate consumption (light gray bar in Figure 4). Second, the decrease in the dispersion of consumption provides an additional push to inflation (dark gray bar). Third, these two factors trump the lower increase in hours (red bar).

The differential responses of consumption and employment in these two models are a consequence of the differences in shock propagation when markets are incomplete. In the RANK model, intertemporal substitution leads households to reduce consumption and work more to accumulate more assets when real rates are increasing. In the HANK model, however, there is an additional income effect as households can self-insure better (there are now more bonds), which leads to higher consumption (everything else equal).



(a) HANK model

(b) RANK model

Figure 5: Decomposition of the response of aggregate consumption

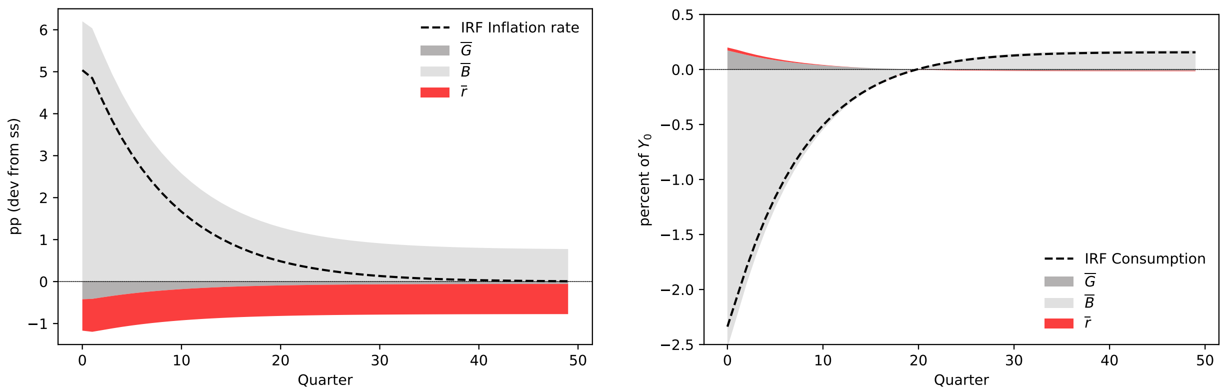
Note: The decomposition uses the Jacobians of the household block in each model and then combines them with the general equilibrium response of real after-tax income and the ex-post real interest rate in each model. The decomposition is exact for linear IRFs. The vertical axis is the same in both graphs.

Figure 5 decomposes the aggregate response of consumption into the impact of real interest rates (which lumps together intertemporal substitution and income effects linked to real rates) and after-tax labor income. It shows how even if the impact of real rates is stronger in HANK, which would imply a lower path of consumption, it is more than compensated by the income effect in the short and medium run. In the long run, the higher income due to higher real rates dominates. The response in hours is the opposite of that in consumption, as real wages remain constant.

response of aggregate consumption C_t, a consumption heterogeneity measure $\int u'(c_{i,t}) z_{it} di - u'(C_t)$, and employment N_t in each model. The impact of aggregate consumption for the HANK model is obtained by replacing the average of marginal utilities of consumption with the marginal utility of aggregate consumption in the wage Phillips curve. The part explained by heterogeneity is the difference between these two measures.

5.3 Decomposing the changes in policy

As we described above, our policy experiment considers that three policy parameters change on impact: the debt target $\overline{B}$, the expenditure target $\overline{G}$, and third, the intercept in the monetary policy rule $\overline{r}$. The last two changes are necessary to ensure the long-run stability of the model. Nonetheless, it is important to understand the role played by each change separately.



(a) Inflation

(b) Consumption

Figure 6: Decomposition of the response of inflation and consumption in terms of policy variables

Panel (a) in Figure 6 decomposes the response of inflation into the individual effects of each of these changes. The decomposition separates the impulse response function (IRF) of inflation into these three components using the sequence-space representation of the model, following the methodology of Auclert et al. (2021a). To a first-order approximation, the general equilibrium response of inflation can be decomposed as $d\pi = \mathcal{J}_{\pi,\overline{G}} \times d\overline{G} + \mathcal{J}_{\pi,\overline{B}} \times d\overline{B} + \mathcal{J}_{\pi,\overline{r}} \times d\overline{r}$, where $\mathcal{J}_{\pi,x}$ is a matrix containing the Jacobians of inflation with respect to the three policy variables $x = [\overline{G}, \overline{B}, \overline{r}]$ at different horizons and dx are column vectors that contain the complete temporal paths followed by these three variables in their transition to the new DSS.

Three important results emerge from this exercise. First, the bulk of the inflationary response is due to the fiscal expansion (light gray area). We discuss the particular mechanisms behind this result below.

Second, the impact of the expenditure target $\overline{G}$ only shows up in the short run: if it did not change, the level of long-term debt would be below $\overline{B}$ to guarantee debt sustainability (that is the opposite of the dark gray area). In any case, its impact is relatively small.

Third, and the most relevant for the question in this paper, not updating the monetary policy rule to the new natural rate would have important inflationary effects. This can be seen by the red area in Panel (a). It shows how the deflationary impact of changing $\overline{r}$ is around

0.7 p.p. in the long run and, interestingly, in the short run too. In other terms, if the central bank sticks to the *old* monetary policy rule and does not update it, inflation would be 0.7 p.p. higher. This number is consistent with the analytical formula (5) for the DSS. The difference between the natural rate and the intercept in the monetary policy rule $r^* - \bar{r}$ is 18 b.p., and the slope ϕ_π is 1.25, the increase in steady-state inflation would be then $0.18 \times 4 = 0.7$ p.p.

In Panel (b), we apply the same decomposition to the consumption response. In this case, as in the rest of the real variables, including real interest rates (not shown), the intercept $\bar{r}$ plays a negligible role. The conclusion is that while $\bar{r}$ is a significant driver of inflation dynamics, it does not affect real variables.

5.4 Nonlinearities

Figure 1 shows how the steady-state mapping from debt targets to natural rates is nonlinear. To see this, notice that the mapping is just given by the demand for bonds, which is indeed nonlinear. While the nonlinearity is stronger for relatively low debt levels, it is still present in our calibration. Results presented in Table 3 in Appendix D show the main variables in the new DSS reached by a 10 p.p. increase in the debt target and those in the case of a 10 p.p. decline. Clearly, the new steady states are asymmetric. Similarly, Table 4 in Appendix D shows the asymmetric response one quarter after the announcement ($t = 2$) (we select one quarter to avoid the effects associated with the Fisher effect on impact). Nonlinearities are smaller in the short run than in the long run, but they are still present.

6 Extensions

Next, we present several extensions of our analysis. Among the many possible exercises of interest, we will focus on the role of anticipated effects of changes in fiscal policy, the possibility of designing robust monetary policy rules and introducing long-term bonds.

6.1 Anticipated Effects

So far, we have assumed that the central updates its monetary policy rule at impact in response to the fiscal shock. We now explore the consequences of changes in the timing of the update.

To this end, Figure 7 shows the results of simulations in which the treasury announces at time zero a debt-financed fiscal expansion that will start 12 quarters in the future. That is, $\overline{B}$ in the fiscal rule changes at time $t = 12$, but this change is known by all agents at time $t = 0$.

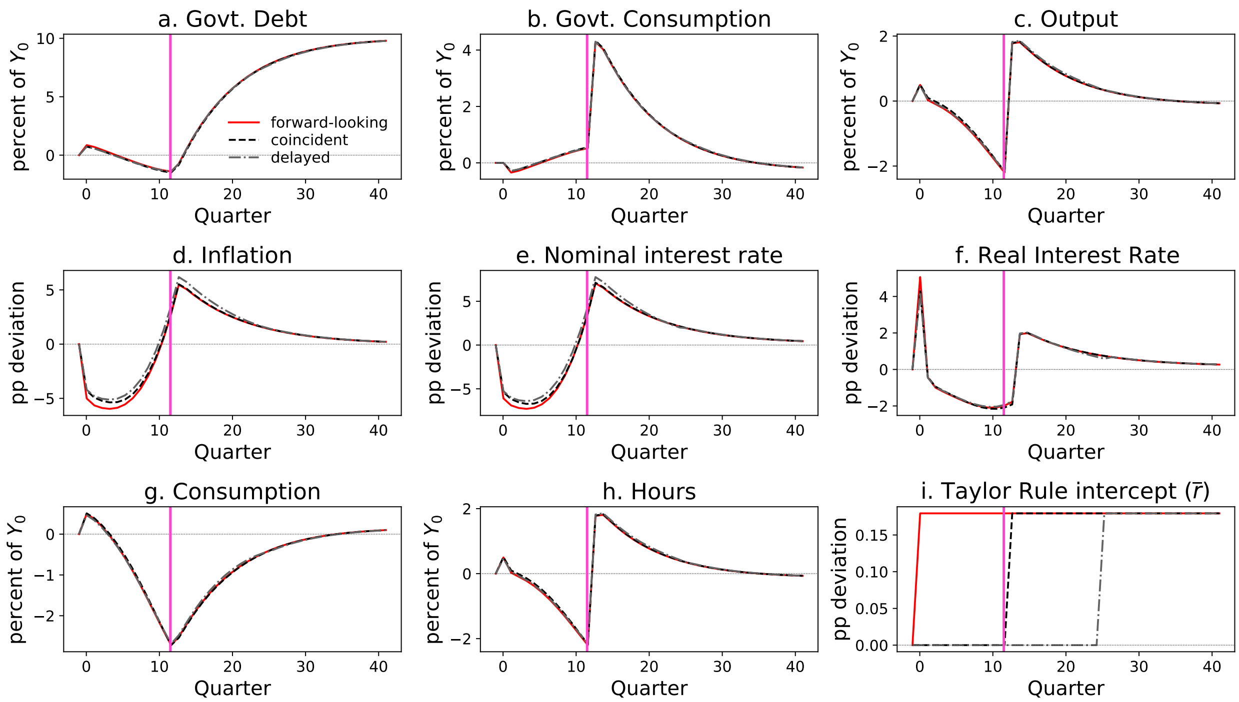


Figure 7: Dynamics of an anticipated debt-financed fiscal expansion

Note: Deviation with respect to the initial DSS after a shock that increases the value of the debt target $\overline{B}$ in the future, after 12 quarters. The figures compare three cases. In the first case, the central bank reacts immediately, anticipating fiscal policy (solid red line). In the second case, it reacts after 12 quarters (dashed black line), at the same time as fiscal policy changes. In the last one, it delays the monetary policy response by another 12 quarters (dotted dashed gray line)). Agents have perfect foresight in all cases.

We start with a scenario in which the central bank updates the intercept in the monetary policy rule $\overline{r}$ as soon as the announcement is made (solid red line). The dynamics after the actual arrival of the shock are similar to those in the baseline model described in figure 3. Inflation increases by 5 p.p., GDP increases by 1.5 p.p., and real rates by 2 p.p.. What is new are the dynamics *before* the actual change in $\overline{B}$. As households are anticipating higher real rates, they start saving in advance, which explains the downward trend in consumption. The increase in the demand for savings reduces real interest rates and has deflationary effects. The lower real rates reduce the fiscal burden, which allows for both a persistent reduction in the stock debt and an increase in government spending. The first period deserves special attention. The plunge in inflation on impact implies a decrease in the price level, which leads to a sudden increase in the real value of debt through the Fisher effect. The counterpart to this is the initial increase in consumption due to the rise in real wealth. But, given that bonds are one-period, this effect is short-lived (we will revisit this issue in Section 6).

Next, we consider the case in which the central bank decides to maintain its monetary policy rule constant until the *actual change* in $\overline{B}$ takes place (dashed black line). This implies that starting in period $t = 12$, the rule coincides with the one just described. The difference between them is just limited to the first 11 quarters. This helps to explain why the dynamics are almost identical after the fiscal expansion actually takes place. The main difference lies in the period of divergence, in which the non-anticipatory rule displays a less marked deflationary path and higher nominal rate. Notice that the initial higher nominal rate path emerges despite the fact that $\overline{r}$ is lower in this case. The path of real variables is almost identical in both cases, in line with the result discussed in Figure 6 above.

We consider last the case of a *delayed* change in the rule, which happens another 12 quarters after the change in the debt target $\overline{B}$. As we observed above, the dynamics of inflation are identical after $t = 24$, but this delayed case exhibits higher inflation and nominal rates compared to the previous rules. The path of real rates and real variables is again indistinguishable from the two-period cases.

We draw some conclusions from this exercise. First, the more delayed the update in the intercept is, the higher the inflation response. Second, once the update happens, the dynamics are similar to those in the baseline. Third, and in line with previous results, the intercept affects inflation and nominal rates, but it is irrelevant for real variables.

6.2 Robust monetary rules

We have seen so far how the central bank's endogenous reaction to fiscal changes affecting the natural rate plays a significant role in inflation dynamics. If those changes are relatively frequent, the central bank would be forced to update its policy rule regularly.

One alternative would be to find a monetary policy rule that does not require any knowledge regarding the value of the natural rate. Such a rule was proposed by Orphanides and Williams (2002), and it can trace its origin to early work by Phillips (1954). The idea is to link the change in nominal interest rates $i_t - i_{t-1}$ to the deviation of inflation from its target $\pi_t - \overline{\pi}$. The rule can be written as

$$\log(1 + i_t) = \log(1 + i_{t-1}) + \phi_\pi \log\left(\frac{1 + \pi_t}{1 + \overline{\pi}}\right). \tag{6}$$

The advantage of the rule (6) is that the natural rate does not appear explicitly. The disadvantage may precisely be that the central bank is renouncing to use of valuable information about the natural rate when formulating it.

Figure 8 compares this robust Orphanides-Williams (OW, dashed black line) with the baseline Taylor rule (solid red line) in which the intercept $\overline{r}$ is updated on impact. The experiment is the same as above, namely a 10 p.p. increase in the debt target $\overline{r}$ (with an

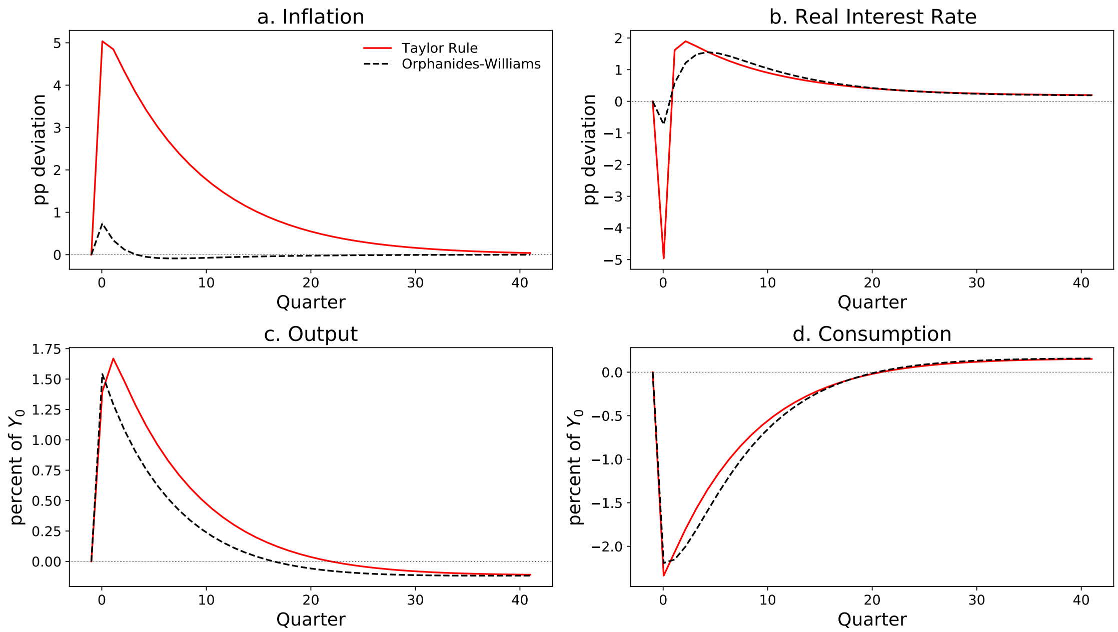


Figure 8: Comparison of a standard Taylor Rule and the Orphanides-Williams Rule in the HANK Model

Note: Deviation with respect to the initial DSS.

associated change in $\overline{B}$).

Both rules successfully converge to zero inflation in the new DSS. In the case of OW, the steady-state Taylor rule implies $\pi_{ss} = \overline{\pi}$, that is, if an equilibrium exists, it implies a steady state in which the inflation target is achieved. Notice that, for clarity, we are abstracting here from the ZLB, the two inflationary regimes, and the minimum debt level compatible with the inflation target.

The OW rule with the same slope ϕ_π reduces the volatility of both inflation and output (Panels a and c) compared to the Taylor rule at the cost of a marginally more persistent decline in consumption.

In order to interpret those results, we can express the OW rule recursively as:

$$\log(1+i_t) = \log(1+i_0) + \sum_{i=0}^{t-1}\left[\phi_\pi \log\left(\frac{1+\pi_{t-i}}{1+\overline{\pi}}\right)\right],$$

where the initial nominal rate is given by $\log(1+i_0) = \log\left[(1+\bar{r})(1+\overline{\pi})\right]$ and $\bar{r}$ is the intercept of the Taylor rule before the fiscal announcement, which coincides with the natural rate in the old steady state. The ex-post real interest rate is thus

$$\log(1+r_t) = \log(1+\bar{r}) + (\phi_\pi - 1)\log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right) + \sum_{i=1}^{t-1}\left[\phi_\pi \log\left(\frac{1+\pi_{t-i}}{1+\overline{\pi}}\right)\right],$$

whereas, in the baseline Taylor rule, it was

$$\log\left(1+r_t\right)=\log\left(1+r^*\right)+\left(\phi_\pi-1\right)\log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right).$$

In this equation, we have used the fact that the central bank updates the intercept of the Taylor rule to r^*.

The comparison between the two rules makes evident the differences between them: while the Taylor rule focuses on contemporaneous inflation deviations, the OW rule includes an additional term that averages all past inflation deviations. That makes the real rate a slow-moving object, more persistent than under the Taylor rule (Panel b), which explains the dampened reaction of inflation. Even if the central bank does not explicitly target the new natural rate, it "learns it" by accumulating inflation deviations.

6.3 Introducing long-term debt

In our final extension, we replace one-period bonds with long-term bonds that pay a geometrically decaying coupon. This is a common modeling choice (e.g., Woodford, 2001). Long-term bonds B^L pay a nominal dividend of 1 in the first period, δ in the second period, δ^2 in the third period, and so on, with $0 \leq \delta < 1$. If $\delta = 0$, then long-term debt is effectively one-period debt. Arbitrage considerations imply that in a rational expectations equilibrium, the short-term ex-ante nominal interest rate must satisfy:

$$1+i_t=\frac{1+\delta Q_{t+1}}{Q_t}.$$

When the treasury borrows using long-term bonds, the treasury budget constraint (2) becomes:

$$P_tQ_tB_t^L=(1+\delta Q_t)P_{t-1}B_{t-1}^L+P_t(G_t-T_t),$$

where the left-hand side is the nominal value of the stock of bonds at the end of period t, and the right-hand side is the nominal value of bonds at the start of period t plus the dividends and the primary deficit.

We define the value of bonds expressed in date-t consumption goods (the real market value of debt) as $\tilde{B}_t \equiv Q_tB_t^L$. Then, $\tilde{B}_t=(1+r_t)\tilde{B}_{t-1}+G_t-T_t$, with the ex-post real return on bonds given by $1+r_t \equiv \frac{(1+\delta Q_t)}{Q_{t-1}}\frac{P_{t-1}}{P_t}=(1-i_{t-1})\frac{P_{t-1}}{P_t}$.[9] Market clearing with long-term bonds

[9] This ex-post real return will not coincide with the ex-ante real return that was expected by bondholders in the period $t-1$ prior to the arrival of an unexpected shock. The reason is twofold. First, the price level in period t will jump, as occurs with short-term debt. Second, Q_t will also jump, leading to a discrepancy between the nominal return of the bond that was expected in the prior period and the realized ex-post nominal

requires that:

$$\int_0^1 a_{it} di = Q_t B_t^L = \tilde{B}_t,$$

that is, household wealth is now saved in the form of long-term debt.

We modify the fiscal rule so that it reacts to the real market value of debt $\tilde{B}$:

$$G_t = \overline{G} - \phi_G(\tilde{B}_{t-1} - \overline{B})$$

We choose the same parameter values for $\overline{G}$ and $\overline{B}$ as in the model with one-period debt. This implies that $\tilde{B}_{ss} = Q_{ss} B_{ss}^L = \overline{B}$. From the market clearing condition in the financial market, we again have that $A_{ss} = \overline{B}$ and the steady-state one-period real interest rate is, therefore, the same as in the model with one-period public debt. We calibrate the parameter δ to 0.95 so that the steady-state duration of bonds is 18 quarters (4.5 years), similar to the duration of assets and liabilities in the U.S. economy estimated by Doepke and Schneider (2006b).[10]

The introduction of long-term bonds has two main consequences. First, it greatly amplifies the (transitory) responses of nominal and real variables to a debt-financed fiscal expansion. Second, it modifies the result, discussed in the context of Figure 6, that if the central bank sticks to the old monetary policy rule, it affects inflation but not real variables such as consumption or output. With long-term debt, instead, real variables are also significantly affected along the transition path.

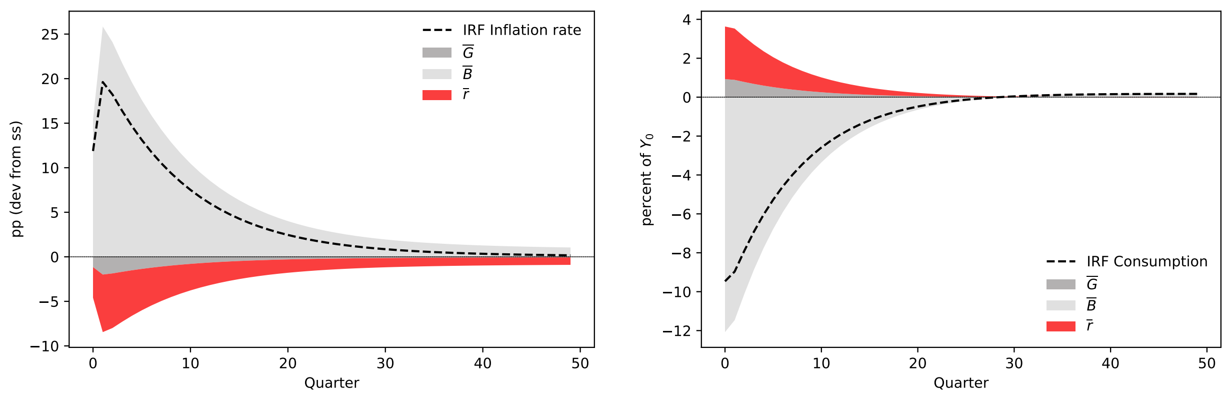
Both consequences are due to the Fisher effect: with long-term debt, the previously unanticipated increase in the path of inflation following the fiscal announcement reduces the real price of bonds Q at time zero. That implies that the real market value of debt plunges on impact, reducing the fiscal burden of the treasury. Given the fiscal rule, that allows the treasury to engage in a much larger fiscal expansion than with short-term debt.[11] Interestingly, this mechanism implies a larger wealth redistribution from wealthy households, who initially owned most of the bonds, to the treasury.

Figure 9 displays the decomposition of the response of inflation and consumption into the individual effects of changes in the debt target $\overline{B}$, the expenditure target $\overline{G}$, and the intercept in the monetary policy rule $\overline{r}$. Contrary to what happened in Figure 6, the intercept now plays a significant positive role in the consumption response in the first 20 quarters. If the central bank did not adjust its rule, that would lead to higher inflation and, thus, to even lower initial market values of debt and an even larger fiscal expansion. This channel explains its influence on the consumption response.

return of holding the bond.

[10]The formula for the duration is $D_{ss} = (1 + i_{ss})/(1 + i_{ss} - \delta)$.

[11]The response is qualitatively similar to that displayed in Figure 3, but quantitatively much larger. The key difference is the decline in the stock of debt on impact, as displayed in Figure 14 in Appendix D.



(a) Inflation (b) Consumption

Figure 9: Decomposition of the response of inflation and consumption in terms of policy variables with long-term bonds

7 Validating evidence

In this section, we provide empirical support to validate the two main claims of the paper: (i) that debt-financed fiscal expansions increase the natural rate; and (ii) that central banks adjust their reaction function in response to high- and medium-frequency changes in natural rates.

7.1 Response of the natural rate to an increase in public debt

We want to estimate the IRF of the natural rate, r^*, to an increase in the debt-to-GDP ratio. As our measure of r^*, we rely on the natural rate estimated by Lubik and Matthes (2015) using a time-varying parameter vector autoregressive model (TVP-VAR).[12]

We tackle the estimation from two complementary approaches. In our first approach, we estimate the IRF using local projections (LP; Jordà, 2005). More concretely, we specify:

$$r^*_{t+h} = \alpha_h + \beta_h D_{t-1} + \mathbf{x}_t \gamma_h + u_{t+h}$$

for $h = 0, \ldots, H$, where r^*_{t+h} is the outcome variable, the natural rate r^* observed h periods from today, D_{t-1} refers to the lagged debt-to-GDP ratio, β_h is the IRF of the outcome variable at horizon h relative to its lagged value today; $\mathbf{x}_t$ collects all additional control including lags of the outcome variable, and the debt-to-GDP ratio, as well as lagged values of federal funds rate, inflation, and the unemployment rate. The estimation sample spans the period between the first quarter of 1967 and the second quarter of 2023.

[12] A continuously updated data-set is available at `https://www.richmondfed.org/research/national_economy/natural_rate_interest`.

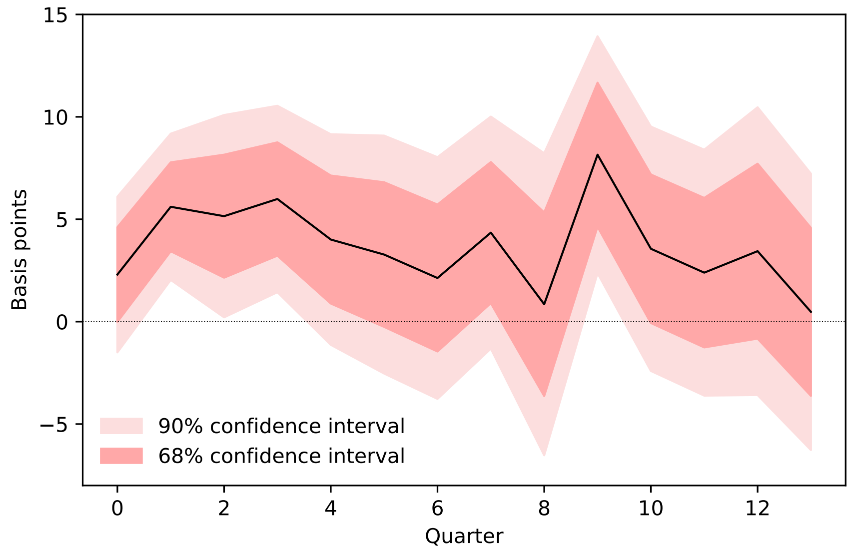


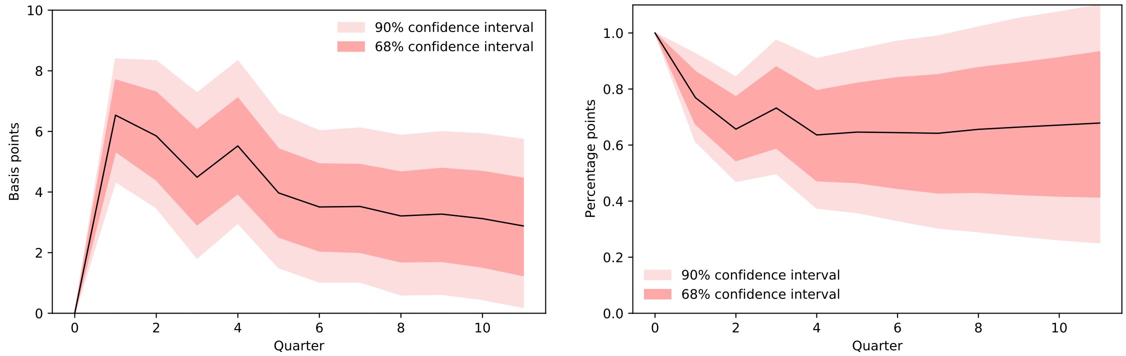
Figure 10: IRF of r^* to a 1% increase in the public debt-to-GDP ratio, LP

Note: We estimate a local projection with $r^*_{t+h} = \alpha_h + \beta_h D_{t-1} + \mathbf{x}_t \gamma_h + u_{t+h}$ and plot the regression coefficient β_h (the solid line) associated to the lagged public debt-to-GDP ratio D_{t-1}. The control variables $\mathbf{x}_t$ include four lags of the change of r^*, three additional lags of the public debt-to-GDP ratio, and four lags of the federal funds rate, the GDP deflator, and the unemployment rate. The shaded areas represent the 68% and 90% confidence intervals using Eicker–Huber–White standard errors following Montiel Olea and Plagborg-Møller (2021).

Figure 10 shows the IRF of r^* to 1 p.p. change in the public debt-to-GDP ratio. We estimate that r^* increases by between 2 and 5 b.p. immediately after a 1 p.p. shock. This is equivalent to 20-50 b.p. in response to a 10 p.p. fiscal shock. Recall that our model delivers, for a 10 p.p. fiscal shock, a change in r^* of 18 b.p. (Table 2). Thus, our model roughly matches the lower range of our empirical estimates. Our empirical estimate is also similar to the results in Rachel and Summers (2019, Table 2). Averaging the findings in the literature, these authors report that an increase of 10 p.p. in the debt-to-GDP ratio leads to an increase in r^* of 35 b.p.

In our second approach, we estimate a structural vector autoregression (SVAR) model with r^*, the debt-to-GDP ratio, the federal funds rate, inflation, and the unemployment rate as variables. We identify a structural shock using a recursive (Cholesky) identification, assuming that r^* responds to the lagged debt-to-GDP ratio, which is consistent with the specification used in the exercise using local projections and with an interpretation of our model where the central bank has a fast but not instantaneous response to the fiscal shock (perhaps due to delays in the reporting of fiscal spending and tax revenues).

Figure 11 plots the response to a structural shock that leads to a one-point increase in the debt-to-GDP ratio. The findings indicate similar effects on r^*, ranging between 3 and 6 b.p. Moreover, the impact on debt level appears to be highly persistent, in line with the theoretical



(a) Response of r^*

(b) Response of debt as percentage of GDP

Figure 11: IRF of r^* to a 1% increase in the public debt-to-GDP ratio, SVAR

Note: IRF of r^* and the debt-to-GDP ratio to a one point shock to the public debt-to-GDP ratio. We use a Cholesky identification with the order: public debt-to-GDP, r^*, inflation, unemployment rate, and the federal funds rate. The lag length is $p = 4$. The sample ranges from the first quarter of 1967 to the second quarter of 2023. The solid lines are the point estimates. The shaded areas represent the 68% and 90% confidence intervals computed using a wild bootstrap with 10,000 replications.

result that natural rates respond to permanent increases in debt. Again, these results roughly agree with the findings of our model. Even if the latter might be underestimating the total impact of fiscal shocks on r^*, it gets into a reasonable distance of the empirical findings, and in stark comparison with the RANK model, where r^* remains unchanged.

7.2 Natural rates and the central bank's reaction function

We move to explore the empirical link between r^* and the central bank's monetary policy.

Assume that the central bank reaction function can be expressed as

$$i_t = \sum_{k=1}^{K} \rho_k i_{t-k} + \left(1 - \sum_{k=1}^{K} \rho_k\right) \Phi\left(\pi_t, \mathbb{E}_t\left[\pi_{t+1}\right], ..., \mathbb{E}_t\left[\pi_{t+N}\right], \mathbb{E}_t\left[\tilde{y}_{t+1}\right], ..., \mathbb{E}_t\left[\tilde{y}_{t+N}\right]\right),$$

where $\sum_{k=1}^{K} \rho_k i_{t-k}$ is an autoregressive structure that accounts for the potential smoothing in nominal rates and $\Phi(\cdot)$ is a nonlinear function mapping inflation, inflation expectations, output gap $\tilde{y}$, and gap expectations to nominal rates. This is a very general formulation that provides a flexible mapping to the reaction function of central banks in reality.

If we take a first-order Taylor expansion around the central bank target $\overline{\pi}$ and the zero output gap, we can express the policy rule as:

$$i_t \approx \sum_{k=1}^{K} \rho_k i_{t-k} + \Phi_0 + \sum_{n=0}^{N} \left(\frac{\partial \Phi}{\partial \mathbb{E}_t\left[\pi_{t+n}\right]}\right) \left(\mathbb{E}_t\left[\pi_{t+n}\right] - \overline{\pi}\right) + \sum_{n=0}^{N} \left(\frac{\partial \Phi}{\partial \mathbb{E}_t\left[\tilde{y}_{t+n}\right]}\right) \left(\mathbb{E}_t\left[\tilde{y}_{t+n}\right]\right).$$

In steady state, we have that $i_{t-k} = i_{ss}$ for all k, $\mathbb{E}_t\left[\pi_{t+n}\right] = \pi_{ss}$ and $\mathbb{E}_t\left[\tilde{y}_{t+n}\right] = 0$ for all n. Thus, we can express the rule as

$$i_{ss} \approx \bar{r} + \overline{\pi} + \phi_\pi \left(\pi_{ss} - \overline{\pi}\right),$$

where $\phi_\pi = \sum_{n=0}^{N}\left(\frac{\partial \Phi}{\partial \mathbb{E}_t[\pi_{t+n}]}\right)$ and the intercept $\bar{r} = \Phi_0 - \overline{\pi}$. If we substitute the steady-state nominal rate through the Fisher equation, we obtain equation (4) outside of the ZLB:

$$r^* + \pi_{ss} \approx \bar{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}).$$

We can then obtain equation (5)

$$\pi_{ss} \approx \overline{\pi} + \frac{r^* - \bar{r}}{\phi_\pi - 1}.$$

This equation links the deviations of long-term inflation from the central bank target $\pi_{ss} - \overline{\pi}$ with the *policy gap* $r^* - \bar{r}$ between the natural rate and the intercept in the central bank reaction function.

If the steady-state objects inherit the time-varying nature of fiscal shocks, we can think about them as random variables instead of constant parameters. More in concrete, if the intercept $\bar{r}$ perfectly tracks the natural rate r^*, then the policy gap is always zero, and steady-state inflation remains constant at the central bank target $\pi_{ss} = \overline{\pi}$. Alternatively, if the Taylor rule remains stable in the sense that its intercept does not co-move with changes in long-term inflation ($cov\left(\bar{r}, \pi_{ss}\right) = 0$), then we can compute the variance of inflation as

$$var\left(\pi_{ss}\right) \approx \frac{cov\left(r^*, \pi_{ss}\right) - cov\left(\bar{r}, \pi_{ss}\right)}{\phi_\pi - 1} = \frac{cov\left(r^*, \pi_{ss}\right)}{\phi_\pi - 1},$$

where we have assumed that ϕ_π remains constant. In this case, the policy gap can be computed as

$$r^* - \bar{r} = \frac{cov\left(r^*, \pi_{ss}\right)}{var\left(\pi_{ss}\right)}\left(\pi_{ss} - \overline{\pi}\right). \tag{7}$$

Equation (7) provides a measure of the policy gap that can be estimated using market data.

We do so as follows. First, we need data that approximates the (potentially time-varying) steady-state values for real interest rates and inflation. One possibility is to employ market data on long-term interest rates and inflation compensation for the U.S. Here, we drop the natural rate estimates of Lubik and Matthes (2015) we used before because we also need a proxy for long-term inflation. Using market data provides us with a consistent set of variables.

We collect daily data on the 5-year 5-year (5y5y) forward nominal yield. This is a measure of the 5-year yield expected five years ahead, which is commonly used as a proxy for long-term nominal interest rates. For long-term inflation, we employ the 5y5y inflation-linked swaps (ILS). Those are swap contracts that transfer inflation risk from one party to another through an exchange of fixed cash flows. The real interest rate is computed as the difference between the 5y5y nominal rate and the 5y5y ILS.[13]

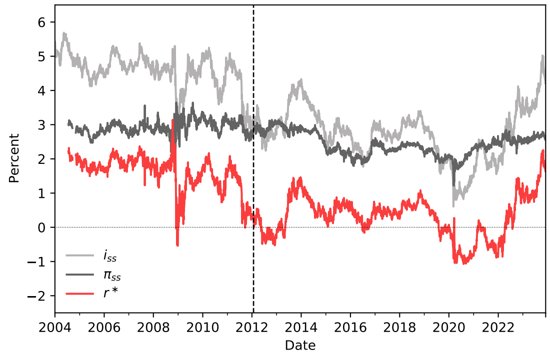
The value of the inflation target $\overline{\pi}$ is set to 2%. The Federal Reserve officially adopted this value in January 2012, but it was considered the implicit target long before that date, in line with other major central banks such as the ECB or the Bank of England. Nonetheless, to be on the safe side, we compute $\frac{cov(r^*,\pi_{ss})}{var(\pi_{ss})}$ only for the period starting in January 2012. The resulting value for this period is 0.56. This implies a value for the Taylor coefficient of $\phi_\pi = 1.56$, which is a standard value in the literature.

Panel (a) of Figure 12 plots the three series since they are available (2004). Two patterns are apparent. First, market expectations of long-term nominal and real rates and inflation are neither constant nor evolve exclusively according to low-frequency secular trends but display a significant level of high- and medium-term volatility. Second, both nominal and real rates display a larger volatility than inflation.

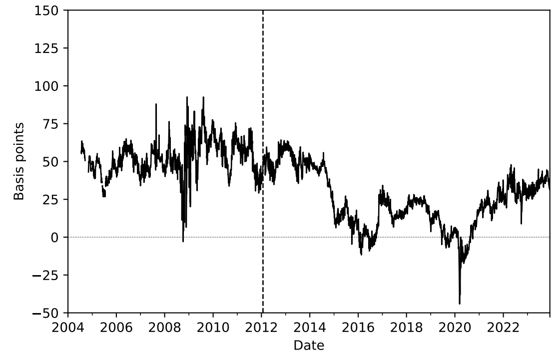
Panel (b) of Figure 12 plots the estimated policy gap. This gap was significantly different from zero before 2014. From 2015 to 2020, the gap largely closed, but it reopened again after the large fiscal expansion that followed the COVID-19 pandemic. These results provide evidence supporting the idea that market participants perceive that the Federal Reserve's reaction function has not always tracked the natural rate perfectly, which explains the dynamics in long-term inflation.

However, the previous series do not reflect genuine market participants' expectations, as they are contaminated by term premia that reflect the compensation that risk-averse investors demand for bearing interest and inflation risk. As a robustness analysis, we remove term premia from the interest rate and inflation data using the methodology described by Hördahl and Tristani (2014). The adjusted series are displayed in Panel (c). Then, in Panel (d), we redo the analysis and display the policy gap estimated using these data. The new series is remarkably similar to the previous one, especially in the post-2012 period. The main difference is due to the higher slope of the Taylor coefficient, $\phi_\pi = 2.63$.

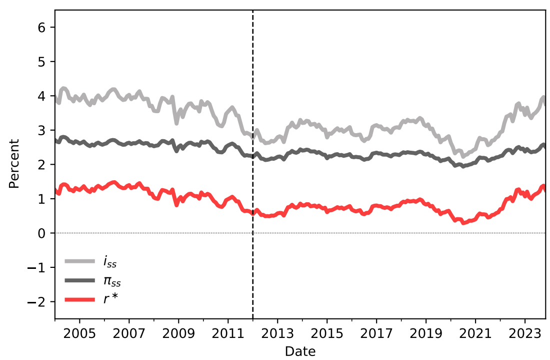
[13]For robustness, we have also employed 5y5y TIPS as a proxy for long-term real rates and computed the break-even inflation rate as the difference between nominal and real rates. The results are quite similar.



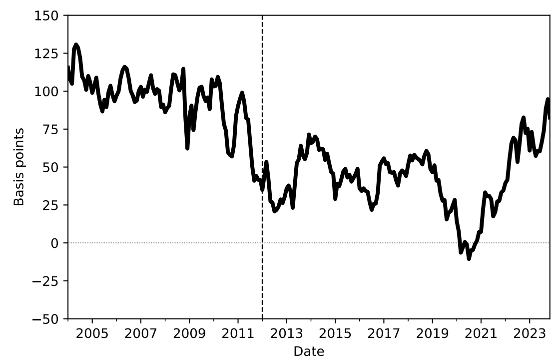
(a) Long-term nominal and real rates and inflation.



(b) Policy gap $r^* - \overline{r}$



(c) Data adjusted for term premia



(d) Policy gap $r^* - \overline{r}$ (adj. data)

Figure 12: Policy gaps

Note: Panel (a): i_{ss} is the 5y5y forward nominal rate obtained from the zero-coupon U.S. yield curve. π_{ss} is the 5y5y ILS. r^* is computed as the difference $i_{ss} - \pi_{ss}$. Panel (b) policy gap is based on Panel (a) data. In Panel (c), the estimated term premia are removed from Panel (a) series using the methodology described by Hördahl and Tristani (2014). The policy gap in Panel (d) is based on Panel (c) data. The dashed vertical line marks the date when the 2% inflation target was announced (January 25, 2012).

8 Conclusions

This paper analyzes a novel type of monetary-fiscal interaction in a HANK model with a fiscal block. In our economy, the stock of public debt affects the natural interest rate, forcing the central bank to adapt its monetary policy rule to the fiscal stance in order to guarantee price stability. We show that there is a threshold of minimal debt, below which the steady-state inflation deviates from its target as the ZLB constraint binds.

We also analyze the response to a debt-financed fiscal expansion and quantify the impact of different timings in the adaptation of the monetary policy rule, as well as the performance of alternative monetary policy rules à la Orphanides and Williams (2002) that do not require an assessment of the natural rates.

The data validate the key properties of our model: the reaction of natural rates to fiscal shocks and the subsequent response of central banks to them.

An important question, which we leave unaddressed, is how these new fiscal-monetary interactions would affect the optimal conduct of monetary policy. We leave this for future research.

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Appendix

A Derivation of the nonlinear wage Phillips curve with trend inflation

Each household i supplies a continuum of labor services $n_{i,k,t}$ that are imperfect substitutes. There is a union for each type of labor k. Each union k aggregates the efficient units of work of its members into a union-specific task:

$$N_{k,t} = \int z_{i,t} n_{i,k,t} di.$$

Unions set a common wage per efficiency unit $W_{k,t}$, and their members have to supply labor demanded at that wage. A competitive labor packer then packages these tasks into aggregate employment services using the constant-elasticity-of-substitution technology:

$$N_t = \left(\int N_{k,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} dk \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}.$$

and sells these services to final goods firms at a price W_t. In this setup, all unions choose to set the same wage $W_{k,t} = W_t$ at time t and all households work the same number of hours, equal to $n_{i,t} = N_t$, so efficiency-weighted hours worked $\int z_{i,t} n_{i,t} di$ are also equal to aggregate labor demand N_t.

We assume that there are quadratic utility costs for adjusting the nominal wage $W_{k,t}$ set by union k:

$$\frac{\psi}{2} \log \left(\frac{W_{k,t+\tau}}{W_{k,t+\tau-1}\,(1+\overline{\pi})} \right)^2$$

.

The problem solved by the union at each date t is:

$$\max_{\{W_{k,t+\tau}\}} \sum_{\tau \geq 0} \beta^{t+\tau} \left[\int \left[u(c_{i,t}) - v(n_{i,t}) \right] di - \frac{\psi}{2} \log \left(\frac{W_{k,t+\tau}}{W_{k,t+\tau-1}\,(1+\overline{\pi})} \right)^2 \right]$$

subject to the demand curve

$$N_{k,t} = \left(\frac{W_{kt}}{W_t} \right)^{-\epsilon_w} N_t,$$

where $W_t^{1-\epsilon_w} = \int W_{k,t}^{1-\epsilon_w} dk$. The solution is a wage Phillips curve of the form:

$$\log\left(\frac{1+\pi_t^w}{1+\overline{\pi}}\right) = \kappa_w\left[-\frac{(\epsilon_w-1)}{\epsilon_w}(1-\tau)\frac{W_t}{P_t}\int u'(c_{it})z_{it}di + v'(N_t)\right]N_t$$
$$+\beta\log\left(\frac{1+\pi_{t+1}^w}{1+\overline{\pi}}\right),$$

with slope $\kappa_w \equiv \frac{\epsilon_w}{\psi}$.

B Aggregate saving behavior in a steady state

The consumer problem of our model in a steady state can be shown to be isomorphic to the consumer problem in a typical Bewley-Imhoroğlu-Aiyagari model.

First, notice that the union makes the labor choice on behalf of the households, which take their labor input as given. Moreover, we have assumed a proportional allocation of labor, which implies $n_{it} = N_t = Y_t/\Theta$, where the second equality follows from the functional form of the production function. Profit maximization by firms implies $\frac{W_t}{P_t} = \Theta$.

Taken together, these relations imply that the problem solved by households can be rewritten as

$$\begin{aligned} V(a_{i,t}, z_{i,t}) = \max_{c_{i,t},a_{i,t+1}} & u(c_{i,t}) - v\left(\frac{Y_t}{\Theta}\right) + \beta\mathbb{E}_t[V(a_{i,t+1}, z_{i,t+1})] \\ \text{s.t. } & c_{i,t} + a_{i,t+1} = (1+r_t)a_{i,t} + (1-\tau)Y_t z_{i,t}, \\ & a_{i,t+1} \geq 0. \end{aligned}$$

In a steady state, output is constant at the value Y_{ss} and the consumer problem for $Y_t = Y_{ss}$ can be written as

$$\begin{aligned} V(a_{i,t}, z_{i,t}) = \max_{c_{i,t},a_{i,t+1}} & \tilde{u}(c_{it}) + \beta\mathbb{E}_t[V(a_{i,t+1}, z_{i,t+1})] \\ \text{s.t. } & c_{i,t} + a_{i,t+1} = (1+r_t)a_{i,t} + \tilde{w}z_{i,t}, \\ & a_{i,t+1} \geq 0, \end{aligned}$$

with $\tilde{u}(c_{it}) \equiv u(c_{i,t}) - v\left(\frac{Y_{ss}}{\Theta}\right)$ and $\tilde{w} \equiv (1-\tau)Y_{ss}$, which is a constant. Written in this form, the optimization problem is identical to the one that appears in equations (1a) and (1b) in Aiyagari (1993) and Aiyagari (1994). Thus, under regularity conditions described in Aiyagari (1993), there exists a unique invariant distribution over assets associated with a steady state, and this invariant distribution behaves continuously with respect to the parameters r^* and $\tilde{w}$ (Aiyagari, 1993, Proposition 5). Moreover, aggregate savings A_{ss} satisfy $\lim_{r*\to\frac{1-\beta}{\beta}} A_{ss}(r^*) = +\infty$ and $\lim_{r*\to-\infty} A_{ss}(r^*) = 0$.

C Existence and uniqueness

Here, we show the existence and uniqueness of a steady state in our model.

Existence. Start from the debt accumulation equation

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t$$

Now substitute G_t with the fiscal rule. This leads to

$$\begin{aligned} B_t &= (1 + r_t)B_{t-1} + \overline{G} - \phi_G(B_{t-1} - \overline{B}) - T_t \\ &= (1 + r_t - \phi_G)B_{t-1} + \overline{G} + \phi_G\overline{B} - T_t \end{aligned}$$

In a steady state,

$$\begin{aligned} B_{ss} &= (1 + r^* - \phi_G)B_{ss} + \overline{G} + \phi_G\overline{B} - T_{ss} \\ &= (1 + r^* - \phi_G)B_{ss} + \overline{G} + \phi_G\overline{B} - (G_{ss} + r^*B_{ss}) \\ &= (1 - \phi_G)B_{ss} + \overline{G} + \phi_G\overline{B} - G_{ss}, \end{aligned}$$

or:

$$B_{ss} = \frac{(\overline{G} - G_{ss})}{\phi_G} + \overline{B}.$$

If $G_{ss} = \overline{G}$, then $B_{ss} = \overline{B}$. Hence, a steady state with $B_{ss} = \overline{B}$ and $G_{ss} = \overline{G}$ exists. These values are independent of the level of the interest rate and a steady state with $B_{ss} = \overline{B}$ and $G_{ss} = \overline{G}$ exists for any value that r^* may take.

Uniqueness. We show next how, as long as ϕ_G exceeds the rate of time preference, i.e., $(1-\beta)/\beta < \phi_G$, the system of equations consisting of the fiscal rule together with the condition for no debt accumulation in steady state can have at most one solution in the (B_{ss}, G_{ss}) space. In fact, it will have exactly one without further constraints, and at most one if an additional constraint, like $B_{ss} \geq 0$, is imposed.

The fiscal rule in steady state and no-debt-accumulation evaluated at steady state values imply

$$G_{ss} = \overline{G} - \phi_G(B_{ss} - \overline{B}) = (\overline{G} + \phi_G \overline{B}) - \phi_G B_{ss}$$
$$= T_{ss} - r^* B_{ss} = \tau Y_{ss} - r^* B_{ss}.$$

These two equations are linear relations between G_{ss} and B_{ss} with intercepts $(\overline{G} + \phi_G \overline{B})$ and τY_{ss} and slopes $-\phi_G$ and $-r^*$, respectively. For any $r^* \leq (1-\beta)/\beta < \phi_G$, they can cross at most once in the (B_{ss}, G_{ss}) plane. Taken by itself, this crossing point need not be the same for all r^*. However, combined with the prior result, there exists a steady state equilibrium located at the point $(\overline{B}, \overline{G})$ regardless of the interest rate, the uniqueness result implies that $(B_{ss}, G_{ss}) = (\overline{B}, \overline{G})$.

Notice also that the intercept τY_{ss} is itself a function of the real interest rate. This does not change the conclusion at which we have arrived because, for any value of Y_{ss}, there can be at most one crossing point in the (B_{ss}, G_{ss}) plane. Finally, because the demand for bonds from households is upward-sloping, there is a unique real interest rate r^* associated with $B_{ss} = \overline{B}$, i.e., for which $A_{ss}(r^*) = \overline{B}$.

D Additional tables and figures

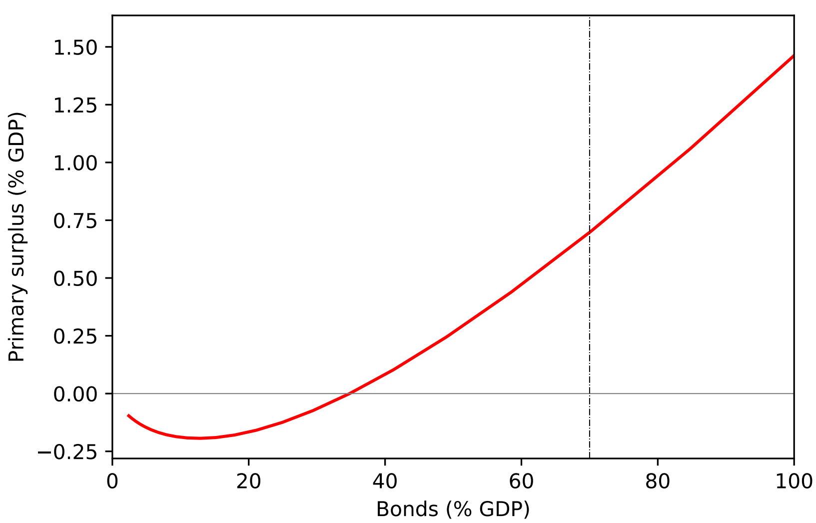

Figure 13: Equilibrium primary surplus

Note: The graphs use the baseline calibration but allow the level of bonds to vary.

	HANK		RANK	
	Expansionary	Contractionary	Expansionary	Contractionary
Bonds (% GDP)	10.000	-10.000	10.000	-10.000
Real interest rate	0.181	-0.214	0.000	0.000
Nominal interest rate	0.205	-0.198	0.000	0.000
Inflation	0.000	0.000	0.000	0.000
Output	-0.112	0.111	-0.042	0.042
Consumption	0.154	-0.139	0.067	-0.067
Govt. consumption	-0.266	0.250	-0.108	0.108
Tax revenue	-0.023	0.023	-0.009	0.009
Primary surplus (% GDP)	0.244	-0.228	0.100	-0.100

Table 3: Asymmetric effects of expansionary and contractionary fiscal policy: steady-state values

	HANK		RANK	
	Expansionary	Contractionary	Expansionary	Contractionary
Bonds (% GDP)	2.208	-2.190	2.293	-2.288
Real interest rate	1.898	-2.033	1.178	-1.120
Nominal interest rate	5.613	-6.046	3.687	-3.504
Inflation	4.328	-4.663	2.939	-2.799
Output	1.484	-1.487	1.589	-1.562
Consumption	-1.800	1.829	-1.775	1.797
Govt. consumption	3.284	-3.316	3.363	-3.358
Tax revenue	0.307	-0.308	0.329	-0.323
Primary surplus (% GDP)	-2.977	3.008	-3.035	3.035

Table 4: Asymmetric effects of expansionary and contractionary fiscal policy: period $t = 2$

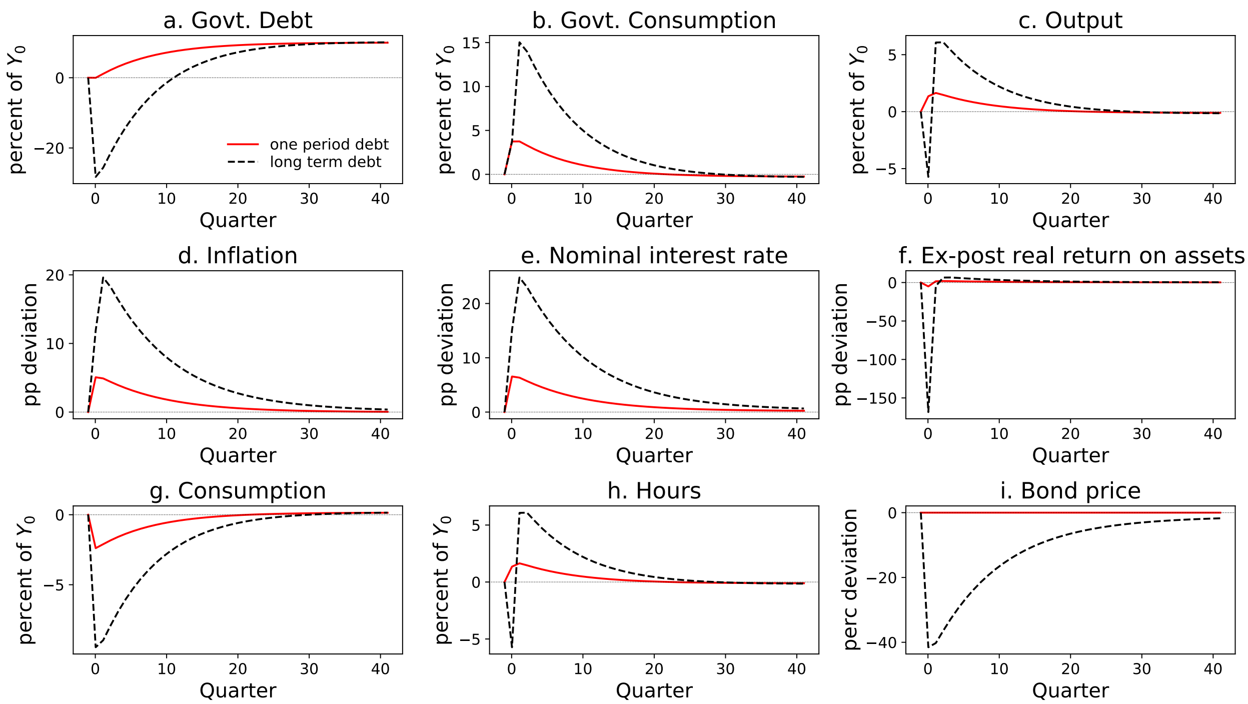

Figure 14: Dynamics of a debt-financed fiscal expansion with long-term debt
Note: Deviation with respect to the initial DSS after a shock that increases the value of the debt target $\overline{B}$ by 10% of initial GDP. Interest rates, inflation, and the ex-post real return on assets are annualized.