American Economic Journal: Macroeconomics 2017, 9(2): 32–72
https://doi.org/10.1257/mac.20140084

Bank Leverage Cycles†

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We propose a general equilibrium framework with financial intermediaries subject to endogenous leverage constraints, and assess its ability to explain the observed fluctuations in intermediary leverage and real economic activity. In the model, intermediaries ("banks") borrow in the form of short-term risky debt. The presence of risk-shifting moral hazard gives rise to a leverage constraint, and creates a link between the volatility in bank asset returns and leverage. Unlike TFP or capital quality shocks, volatility shocks produce empirically plausible fluctuations in bank leverage. The model replicates well the fall in leverage, assets, and GDP during the 2007–2009 financial crisis. (*JEL* D82, E44, G01, G21, G32)

The 2007–2009 financial crisis witnessed a severe disruption of financial intermediation in many industrialized economies. This has led to a surge in both empirical and theoretical research aimed at understanding the causes and consequences of the financial crisis, evaluating the policy measures put in place to tackle its effects, and proposing further policy actions and new regulatory frameworks.

A particularly influential strand of the literature has focused on the role played by the deleveraging of the financial intermediation sector in the propagation of the financial turmoil. Before the crisis, a significant share of financial intermediaries funded their asset purchases primarily by means of collateralized debt with very short maturity, such as sale and repurchase (repo) agreements or asset backed commercial

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† Go to https://doi.org/10.1257/mac.20140084 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

paper (ABCP).[1] As argued by Brunnermeier (2009), Gorton and Metrick (2010, 2012), Krishnamurthy, Nagel, and Orlov (2014), and others, the initial losses suffered by some of the assets that served as collateral in repo or ABCP transactions, together with the uncertainty surrounding individual exposures to such assets, led the holders of that short-term debt (mostly institutional investors, such as money market funds) to largely stop rolling over their lending. This funding freeze forced the financial intermediaries to deleverage, with the resulting contraction in their balance sheets and ultimately in the credit flow to the real economy.

In fact, the observed deleveraging of financial intermediaries during the 2007–2009 financial crisis is not an isolated episode. As documented by Adrian and Shin (2010, 2011), since the 1960s the leverage ratio (i.e., the ratio of total assets to equity capital) of important segments of the financial intermediation sector has exhibited a markedly procyclical pattern, in the sense that expansions (contractions) in balance sheet size have gone hand in hand with increases (decreases) in leverage.[2] Overall, this evidence points to the importance of leverage fluctuations for the cyclical behavior of financial intermediation and real economic activity.

The aim of this paper is to propose a general equilibrium framework with leverage-constrained financial intermediaries that is able to explain the observed fluctuations in intermediary leverage and its comovement with real economic activity. Following much of the theoretical banking literature, financial intermediaries in our framework are subject to moral hazard due to their ability to shift risks.[3] In particular, our risk-shifting moral hazard specification is based on Adrian and Shin's (2014) static, partial equilibrium framework. Our main theoretical contribution is to incorporate their model of endogenous leverage into a fully dynamic, quantitative general equilibrium framework in which intermediaries have an instrumental role in the channeling of funds from savers to borrowers.

In the model, intermediaries ("banks") borrow in the form of short-term risky debt. The source of risk in bank debt is the following. Firms are segmented across "islands" and are hit by island-specific shocks. Some firms are more exposed to island-specific risk than others. Those more exposed also have lower island-specific returns on average, such that financing them is inefficient. Firms can only obtain funds from banks on the same island, as only the latter have the technology to assess their risk exposure. Banks are thus instrumental in channeling funds to firms, but they are also exposed to island-specific risk. A fraction of them declare bankruptcy and default on their debt in each period.

As in Adrian and Shin (2014), banks are affected by risk-shifting moral hazard. Due to limited liability, banks enjoy the upside risk in their assets over and above the face value of their debt, leaving their creditors to bear the downside risk.

[1] This was especially true for the so-called "shadow banking" sector, which comprises those financial intermediaries (investment banks, hedge funds, finance companies, off-balance-sheet investment vehicles, etc.) that have no access to central bank liquidity or public sector credit guarantees, and that are not subject to regulatory capital requirements. See Pozsar et al. (2010) for an in-depth analysis of "shadow banking" in the United States.

[2] This procyclicality has been particularly strong in the case of security brokers and dealers, a category that used to include investment banks.

[3] The risk shifting theory was developed originally by Jensen and Meckling (1976). Risk shifting has a long tradition in the banking literature; see, e.g., Kim and Santomero (1988), Rochet (1992), and Hellmann, Murdock, and Stiglitz (2000). See also Freixas and Rochet (2008, ch. 8 and 9) and the references therein.

This provides banks with an incentive to finance firms that are more exposed to island-specific risk despite having lower expected returns, i.e., to engage in inefficient lending practices. Such an incentive increases with the bank's debt burden relative to the size of its balance sheet. In order to induce banks to invest efficiently, their creditors restrict bank indebtedness to a certain ratio of bank net worth, i.e., they impose a leverage constraint.

We then calibrate our model to the US economy and analyze its ability to replicate the fluctuations in leverage and its comovement with economic activity. In the data, intermediary leverage is characterized by large fluctuations, relative both to GDP and equity capital. Also, leverage is procyclical, not only with respect to assets (as originally documented by Adrian and Shin 2010, 2011), but also with respect to GDP. In the model, we consider three exogenous driving forces: total factor productivity (TFP), capital quality, and time-varying volatility of island-specific shocks. While TFP shocks are fairly standard in business cycle modeling, capital quality shocks have received attention in recent contributions to the macro-finance literature.[4] Likewise, a recent literature argues that exogenous changes in cross-sectional volatility are key in order to understand aggregate fluctuations.[5] Moreover, such changes in volatility can be interpreted as changes in "uncertainty," which as argued before are considered to have played an important role in the recent financial crisis.

Our results show that TFP or capital quality shocks are unable to replicate the volatility and procyclicality of leverage in the data. Intuitively, both types of shock barely affect banks' risk-taking incentives and hence their leverage constraint. On the contrary, shocks to cross-sectional volatility are able to produce fluctuations in leverage that are both realistically large and procyclical with respect to assets and GDP. The mechanism, which we refer to as the "volatility-leverage channel," is as follows. Consider, e.g., an increase in island-specific volatility. Higher uncertainty regarding asset returns, coupled with limited liability, makes it more attractive for banks to engage in inefficiently risky lending practices. In order to prevent them from doing so, institutional investors impose a tighter constraint on banks' leverage. For given net worth, this deleveraging forces banks to contract their balance sheets, thus producing a fall in funding to firms. This leads to a fall in capital investment by firms, and in aggregate output. The consequence is a positive comovement between leverage, assets, and GDP.

We also study the model's ability to shed light on the historical evolution of real and financial variables, with special attention to the 2007–2009 financial crisis. The model accounts well for the large and protracted fall in GDP, bank leverage, and bank assets that occurred during that period. In this regard, we find a key role for the increase in volatility that took place during the financial crisis.

[4] See, e.g., Brunnermeier and Sannikov (2014), Dewachter and Wouters (2014), Gertler and Karadi (2011), or Gertler and Kiyotaki (2010).

[5] See, e.g., Bloom (2009); Bloom et al. (2012); Christiano, Motto, and Rostagno (2003, 2014); Arellano, Bai, and Kehoe (2012); Cúrdia (2007); Gilchrist, Sim, and Zakrajšek (2010); Kiley and Sim (2011); and Pinter, Theodoridis, and Yates (2013). Bloom (2009) refers to such disturbances as "uncertainty shocks," whereas Christiano, Motto, and Rostagno (2014) label them "risk shocks." Our specification of cross-sectional volatility shocks is most closely related to Christiano, Motto, and Rostagno (2003, 2014).

Finally, our analysis sheds light on a recent debate regarding the use of book valuations of equity capital (on which our empirical findings are based) for the purpose of leverage measurement. As argued by Adrian and Shin (2014), book equity, i.e., the difference between the book value of assets and liabilities, is the appropriate notion of equity if one is interested, as we are, in banks' lending decisions.[6] Ideally, however, book equity should be based on marked-to-market asset valuations. As argued by He, Khang, and Krishnamurthy (2010; HKK), the fact that commercial banks do not mark-to-market a significant fraction of their assets raises doubts as to how well actual data reflect leverage in that sector. We address this issue by constructing an empirical proxy for fully marked-to-market (MTM) leverage of US-chartered depository institutions (which basically reflect US-chartered commercial banks). We find that MTM leverage increases during 2008 relative to its precrisis level, due to the fall in asset valuations in that period. However, MTM leverage starts *falling* rapidly from 2008:IV onwards, eventually reaching an even lower level than observed leverage. We also find a positive historical correlation between MTM leverage and GDP. Therefore, while consistent with HKK's claim that commercial banks' MTM leverage would have increased during 2008, our analysis shows that MTM leverage is procyclical too, and in particular that it *fell* in the later stages of the crisis, at a time when GDP was still significantly below trend.

In our model, we distinguish too between marked-to-market book leverage and book leverage as is constructed in the data, i.e., taking only partially into account asset price variations. We find that marked-to-market leverage is *procyclical* too, although to a lesser extent than (the model counterpart of) observed leverage. The reason is that, following, e.g., a contractionary shock (regadless of its type), the resulting drop in asset prices produces a countercyclical, transitory increase in marked-to-market leverage. In the case of volatility shocks, this transitory effect is eventually dominated by the volatility-leverage channel discussed above. Since volatility shocks dominate leverage fluctuations in the model, the end result is that marked-to-market leverage remains procyclical.

Literature Review.—Our paper contributes to the literature on the macroeconomic effects of financial frictions. A recent literature has provided theoretical explanations for the "leverage cycles" discussed above, with contributions by Adrian and Shin (2014); Ashcraft, Gârleanu, and Pedersen (2011); Brunnermeier and Pedersen (2009); Dang, Gorton, and Holmström (2010); Geanakoplos (2010); and Gorton and Ordoñez (2014), among others.[7] Most of these models consider some type of link between changes in "uncertainty," typically defined as changes in the volatility of shocks, and the emergence of these leverage cycles. While these models provide important insights on the equilibrium behavior of leverage, they are primarily aimed at illustrating theoretical mechanisms and are thus mainly qualitative. In particular, most of these papers consider two- or three-period economies, or two-period-lived

[6] An alternative measure of equity is market capitalization, i.e., the market value of banks' traded shares, which as argued by Adrian and Shin (2014) has to do with how much the bank is worth to its stakeholders and thus is more appropriate for analyzing decisions like new equity issuance, corporate takeovers, etc.

[7] Some of these authors focus on the behavior of "margins" or "haircuts" in short-term collateralized debt contracts, which are closely related to the concept of "leverage."

agents in Overlapping Generations (OLG) setups; they also assume a partial equilibrium structure. We build on this literature by analyzing endogenous leverage cycles in a fully dynamic, general equilibrium model that can be compared to aggregate data, and that allows us to study the interaction between the real and financial sides of the economy.

Our paper is also related to a growing literature on the role of financial intermediaries in dynamic, stochastic, general equilibrium (DSGE) models. Early contributions to the macro-finance literature, such as Bernanke, Gertler, and Gilchrist (1999); Kiyotaki and Moore (1997); and Carlstrom and Fuerst (1997), emphasized the importance of financial frictions for the macroeconomy, but largely obviated the role played by financial intermediaries. Since the recent financial crisis, a number of papers study how frictions arising in the financial intermediation sector affect credit flows to the real economy. Christiano, Motto, and Rostagno (2010) model banks that face asymmetric information and agency problems in their lending activities and incur a cost when creating liquid liabilities such as deposits, but do not consider leverage constraints on financial intermediaries.

More closely related is the work of Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). These authors consider banks that borrow in the form of riskless debt, and that are leverage-constrained due to a moral hazard problem stemming from bankers' ability to divert a fraction of deposits for personal use.[8] They focus their discussion on how adverse shocks (e.g., to capital quality) may disrupt credit supply through their effect on bank equity capital, and how unconventional monetary policy interventions can mitigate the effects of such shocks on economic activity. By contrast, we propose a model where intermediaries face leverage constraints that limit their incentives to fund inefficiently risky activities, as in Adrian and Shin (2014), and we then assess its ability to explain the observed fluctuations in intermediary leverage and economic activity. The central feature of our model is the presence of risky debt contracts, limited liability, and the associated risk-shifting incentives, which gives rise to the volatility-leverage channel discussed above. Interestingly, we find that capital quality shocks such as those considered by Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) are unable to explain, in the context of our model, the procyclicality of leverage in the data.

Brunnermeier and Sannikov (2014); He and Krishnamurthy (2013); Boissay, Collard, and Smets (2013); and Li (2013) propose models with financially constrained intermediaries and characterize their dynamics in a fully nonlinear manner. In such frameworks, the presence of occasionally binding constraints gives rise to endogenously time-varying macroeconomic risk, as intermediaries come close to and eventually hit such constraints.[9] This in turn gives rise to important nonlinearities.[10] Instead, we follow Gertler and Karadi (2011) or Gertler and Kiyotaki (2010)

[8] Gertler and Kiyotaki (2010) also consider idiosyncratic bank liquidity shocks (as in Kiyotaki and Moore 2008), which give rise to an interbank market for funding.

[9] In Boissay, Collard, and Smets (2013), financial crises are not due to occasionally binding collateral constraints but to occasional financial market runs.

[10] Dewachter and Wouters (2014) show how to approximate the equilibrium dynamics of models with occasionally binding financial constraints (such as He and Krishnamurthy 2013) using a third-order perturbation approach.

in solving our model around the steady state using a standard first-order perturbation method, which facilitates simulation as well as likelihood-based estimation.

Finally, our paper also contributes to the literature on the macroeconomic effects of volatility shocks in quantitative general equilibrium models.[11] In our model, first moment shocks (TFP and capital quality) and volatility shocks map differently on real and financial variables: while both types of shock produce sizable fluctuations in GDP, investment, etc., only volatility shocks produce large and procyclical fluctuations in bank leverage. In this sense, our analysis points to the usefulness of financial data for disentangling the effects of first and second moment shocks.

The paper proceeds as follows. Section I presents empirical evidence on the cyclical behavior of financial intermediaries' balance-sheet aggregates and GDP in the United States. Section II lays out a general equilibrium model with leverage-constrained intermediaries and volatility shocks. Section III analyzes some of the theoretical properties of the model, including the volatility-leverage channel. Section IV calibrates and simulates the model, assessing its ability to replicate the data. Section V concludes.

I. Bank Leverage Cycles in the US Economy

In this section, we perform a systematic analysis of the cyclical fluctuations in the main balance sheet components of US financial intermediaries, with special attention to the leverage ratio, and their comovement with real economic activity. Our analysis comprises the main subsectors in what Greenlaw et al. (2008) termed the "leveraged sector," including depository intermediaries such as US-chartered commercial banks and savings institutions, as well as non-depository intermediaries such as security brokers and dealers and finance companies.

The balance sheet size of financial intermediaries is the product of two components: equity capital and leverage ratio. We may thus write $TA_t = LEV_t EC_t$, where TA_t and EC_t denote the book value of total assets and equity capital, respectively, and LEV_t represents the leverage ratio. In logs, we have

$$\log(TA_t) = \log(LEV_t) + \log(EC_t). \tag{1}$$

Table 1 displays a number of statistics regarding the cyclical fluctuations in intermediary leverage, equity capital, total assets, and GDP in the United States. Our leverage, equity, and assets series are constructed using data from the US Flow of Funds.[12] Our sample period is 1981:I–2014:II.[13] We consider three leveraged financial subsectors: US-chartered depository institutions (which include US-chartered commercial banks and savings institutions), security brokers and dealers, and finance companies.

[11] See footnote 5 and the references therein. See also Williamson (1987) for an early theoretical analysis of such shocks in a general equilibrium model with financial frictions.

[12] Leverage is total assets divided by equity capital (both in dollars). "Assets" and "Equity" are total assets and equity capital, both deflated by the GDP deflator. All series are from the US Flow of Funds, except real GDP and the GDP deflator, which are from the Bureau of Economic Analysis. See online Data Appendix for further details. Leverage, assets, equity, and GDP have been logged and linearly detrended.

[13] We choose 1981 as a starting date for two reasons. First, it has been widely documented that a structural break in the volatility of US business cycle fluctuations took place in the early 1980s. Second, as documented by Jermann and Quadrini (2006), this period saw major regulatory and structural changes in the US financial system.

Table 1—Business Cycle Statistics, 1981:I–2014:II

	Depository institutions	Security broker/dealers	Finance companies	Aggregate sector
Standard deviations (%)				
Assets	9.73	34.05	21.00	10.57
Leverage	9.24	25.54	12.50	10.00
Equity	8.06	15.35	13.28	5.27
GDP: 4.36				
Correlations				
Leverage—assets	0.64	0.90	0.80	0.87
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Leverage—equity	−0.37	0.35	0.32	−0.15
	(0.0000)	(0.0000)	(0.0001)	(0.0792)
Leverage—GDP	0.37	0.78	0.69	0.70
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
GDP—assets	−0.06	0.89	0.90	0.62
	(0.4625)	(0.0000)	(0.0000)	(0.0000)
GDP—equity	−0.50	0.69	0.78	−0.08
	(0.0000)	(0.0000)	(0.0000)	(0.3817)

Note: All series are logged and linearly detrended; p-values of the test of no correlation against the alternative of nonzero correlation are reported in parentheses.

Source: US Flow of Funds and Bureau of Economic Analsyis. See online Data Appendix for details.

The table reveals two main stylized facts regarding the cyclical fluctuations in the leverage ratio of financial intermediaries. First, leverage is *volatile*. Notice first that, for the two largest subsectors (depository institutions and broker/dealers), leverage fluctuates more than equity capital, the other determinant of balance sheet size. Also, leverage is several times more volatile than GDP. For broker/dealers and finance companies, the volatility of leverage is about six and three times larger than that of GDP, respectively. For depository institutions, the leverage ratio fluctuates comparatively less, although its standard deviation is still about twice that of GDP.

Second, leverage is *procyclical* with respect to GDP. The correlation of the different leverage ratios with GDP range from 0.37 to 0.78, and are all statistically significant. Also, leverage is strongly procyclical with respect to total assets, which confirms the original finding of Adrian and Shin (2010, 2011).[14] As explained by Adrian and Shin (2010), such a strong comovement reveals an active management of leverage as a means of expanding and contracting the size of balance sheets. This clear picture regarding the procyclicality of leverage contrasts with the case of equity, which is countercyclical (both with respect to GDP and assets) for depository institutions and procyclical for non-depository intermediaries.

Our empirical analysis of leveraged financial intermediaries is performed on a sectoral level. Our subsequent theoretical model, however, contains a single leveraged intermediary sector. For the purpose of comparing the model to the data, it

[14] Our treatment of the data differs somewhat from that of Adrian and Shin (2010). They focus on the comovement between the *growth rates* of leverage and *nominal* total assets. Here, we focus on the behavior of *real* total assets, due both to our interest in the comovement of financial variables with real GDP and for consistency with our subsequent theoretical model. Also, we use linear detrending so as to extract the cyclical component of assets and leverage.

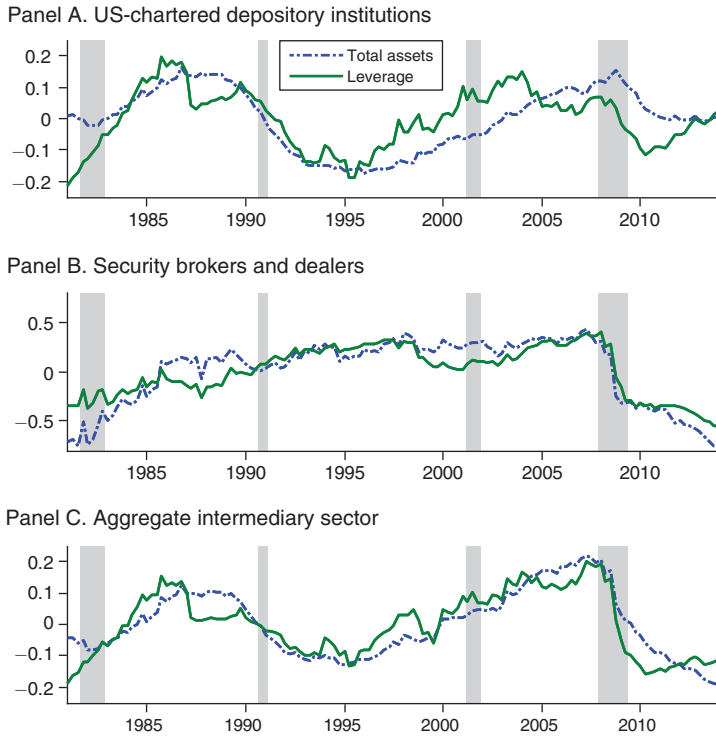

FIGURE 1. CYCLICAL COMPONENTS OF INTERMEDIARY LEVERAGE AND TOTAL ASSETS

Notes: Leverage and total assets have been logged and linearly detrended. Shaded areas represent NBER-dated recessions.

Source: US Flow of Funds. See online Data Appendix for details.

is useful to study the cyclical properties of the leveraged intermediary system as a whole. As a crude approximation of the latter, we construct an "aggregate" leveraged intermediary sector by adding the assets and equity capital of the individual subsectors.[15] The resulting business cycle statistics are displayed in the last column of Table 1. Importantly, the stylized facts of leverage fluctuations mentioned above are preserved for the aggregate sector. The leverage ratio is about two and a half times as volatile as GDP. It is also strongly positively correlated with GDP and assets.

As a graphical illustration, Figure 1 shows the cyclical components of total assets and leverage for the two largest leveraged financial subsectors: US-depository institutions, and security brokers and dealers. It also displays the same series for our measure of the aggregate leveraged intermediary sector. The 2008–2009 recession witnessed a sharp decline in the leverage ratio of security broker/dealers, and a more gradual decline in the leverage ratio of depository institutions that continued after the end of the recession. However, other recessions such as the ones in

[15] A caveat of this approach is that it may lead to double-counting of cross positions between subsectors. However, as discussed before, the cyclical properties of leverage of the different subsectors are similar to each other, and to those of the aggregate sector. Thus the above issue does not seem to be a major concern for the analysis presented here.

1981–1982 or in 2001 did not produce a fall in the leverage of these two subsectors. Thus, intermediary deleveraging seems to have played an important role in some recessions (most notably the Great Recession) but not in others.

The above empirical findings are also robust in other dimensions. First, we have repeated the analysis using a band-pass filter à la Baxter and King (1999) or a Hodrick-Prescott filter instead of linear detrending. Second, we have replaced "total assets" with "total financial assets," which are also available in the Flow of Funds. Finally, we have expanded the sample period by starting it in 1963. In all cases, our results on the procyclicality of leverage and its relative volatility barely change.[16]

To summarize, our empirical analysis suggests the existence of a "bank leverage cycle" in the US economy, characterized by two main features. First, the leverage ratio of financial intermediaries displays large fluctuations, relative both to their equity and to GDP. Second, leverage is procyclical with respect to both assets and GDP. In Section II, we will present a general equilibrium model aimed at explaining these empirical patterns for the aggregate leveraged sector.

On the Measurement of Leverage.—The equity capital series from the Flow of Funds are of *book equity*, i.e., the difference between the value of intermediaries' portfolio of claims and their liabilities. An alternative measure of equity is the market capitalization, i.e., the market value of intermediaries' traded shares. As argued by Adrian and Shin (2014) and Adrian, Colla, and Shin (2013), book equity is the appropriate notion of equity if one is interested in the supply of bank credit, as we are here, whereas market capitalization would be more appropriate if one were interested in new share issuance or mergers and acquisitions decisions.[17]

A related issue is how intermediaries' assets are valued. As stressed by Adrian and Shin (2014), ideally the book value of equity should be *marked-to-market*. This is the case for security broker/dealers or finance companies.[18] As regards depository institutions, He, Khang, and Krishnamurthy (2010; HKK) argue that the fact that part of the assets on commercial banks' balance sheets are not subject to fair value accounting implies that official balance sheet data from the Flow of Funds or Federal Deposit Insurance Corporation (FDIC) may overstate the true procyclicality of book leverage in that subsector.[19] In fact, they argue that, under plausible assumptions on the size

[16] Some of the results can be found in Nuño and Thomas (2013). The rest are available upon request.

[17] As discussed by Adrian and Shin (2014), market capitalization is the discounted value of future free cash flows and therefore includes future cash flows, such as fee income, that do not depend directly on the portfolio held by the bank. Moreover, focus on market capitalization leads to considering the *enterprise value* of the bank, defined as the sum of market capitalization and liabilities. Enterprise value is thus about *how much the bank is worth* to its claim holders, thus making it the appropriate value concept for the purpose of corporate takeovers, sale of new ownership stakes, etc. However, for addressing *how much the bank lends*, the appropriate balance sheet concept is that of total assets. The corresponding equity concept is book equity.

[18] As explained by Adrian and Shin (2014), book equity is fully marked-to-market for financial intermediaries that hold primarily marketable securities.

[19] He, Khang, and Krishnamurthy (2010) argue that, during the 2007–2009 financial crisis, US commercial banks acquired securitized assets from security broker/dealers and other "shadow" intermediaries, and financed these purchases mostly with debt, which would have produced an increase in leverage not reflected in the actual data. Notice that, to the extent the "shadow" intermediaries used the proceeds of such sales to reduce their debt (which is most likely the case), this type of asset redistribution between leveraged subsectors does *not* affect the consolidated leveraged sector or our proxy thereof. As such, they do not affect our subsequent comparison between model and data.

of asset losses not reflected in the data, true leverage would have actually increased during the 2007–2009 financial crisis.

It is difficult to assess quantitatively how relevant this criticism is, given the lack of time series for the commercial banking sector based on marked-to-market valuations. Here we address this issue by constructing a proxy for the *marked-to-market* (MTM) leverage of US-chartered depository institutions (which mostly reflect US-chartered commercial banks). We start from the balance sheet identities $TA_t^{di} = LEV_t^{di} EC_t^{di} = EC_t^{di} + B_t^{di}$, where TA_t^{di} and EC_t^{di} are the observed (book) value of total assets and equity capital of depository institutions, respectively, and $B_t^{di} = TA_t^{di} - EC_t^{di}$ are their non-equity liabilities. We then assume that $TA_t^{di} = \omega_{di}^{MTM} Q_t A_t^{di} + \left(1 - \omega_{di}^{MTM}\right) A_t^{di}$, where A_t^{di} is the quantity of depository institutions' assets, Q_t is an index of bank asset prices, and ω_{di}^{MTM} is the fraction of such assets that are marked-to-market in the books.[20] Our empirical proxy for Q_t is a weighted average of the Barclays Capital US mortgage-backed securities (MBS) and asset-backed securities (ABS) price indexes.[21] As a very conservative estimate, we assume that only one-fourth of all assets of depository institutions are marked-to-market, $\omega_{di}^{MTM} = 1/4$. We can then recover A_t^{di} by computing $A_t^{di} = TA_t^{di} / \left[\omega_{di}^{MTM} Q_t + \left(1 - \omega_{di}^{MTM}\right)\right]$. We finally calculate MTM leverage as $LEV_t^{di,MTM} \equiv Q_t A_t^{di} / \left(Q_t A_t^{di} - B_t^{di}\right)$.

The left panel of Figure 2 compares observed leverage and our MTM leverage proxy for depository institutions during the 2007–2009 financial crisis (the period covered by HKK) and afterwards; the right panel shows our empirical proxy for asset prices. The fall in asset prices that takes place in the second half of 2008 produces an increase in MTM leverage relative to its precrisis level.[22] However, MTM leverage starts *falling* rapidly from 2008:IV onwards, eventually reaching an even lower level than observed leverage. The historical correlation between MTM leverage and GDP is 0.55 and is statistically significant at the 1 percent confidence level (p-value $= 0.0000$).[23] While consistent with HKK's claim that commercial banks' MTM leverage would have increased during 2008, our analysis suggests that MTM leverage is procyclical, and in particular that it *fell* in the later stages of the crisis and afterwards, at a time when GDP was still significantly below trend. Notice finally that while the strong postcrisis recovery in asset prices helps in reducing MTM leverage, this is only part of the story: as shown by the black dashed lines, leverage continues to fall even if one keeps asset prices from 2009:II onwards constant at their previous peak.

The preceding discussion referred to depository institutions. From now onwards, we will compare our single-bank-sector model to the aggregate intermediary sector

[20] We thus assume that, for assets not marked-to-market, banks simply impute a "historical" price of 1.

[21] See the online Data Appendix for further details. The ABS index starts in 1992:I, which is thus the starting date for our asset price series.

[22] The fall in asset prices during the crisis seems relatively small compared to its postcrisis recovery. The reason is twofold. First, both the MBS and ABS indexes already experienced a certain decline in the years before the crisis. Second, the large decline in asset prices during the 2007–2009 financial crisis was concentrated in the ABS segment (which has only a 25 percent weight in our index), where prices fell by almost 20 percent; by contrast, the MBS index did not fall much, possibly reflecting the fact that it covers only agency-backed MBS, which is the least risky segment of the MBS market. See He and Krishnamurthy (2013) for further discussion of the Barclays Capital US MBS index.

[23] The correlation is for the period from 1992:I (the intial date for our asset price series) to 2014:II.

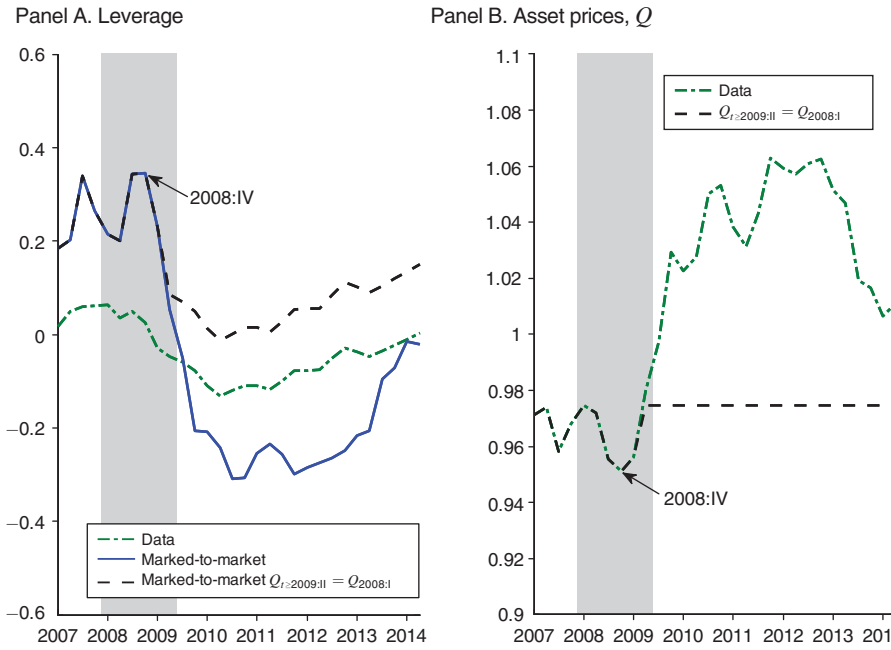

Figure 2. Leverage of US-Chartered Depository Institutions: Data versus Marked-to-Market Proxy

Note: Shaded areas represent NBER-dated recessions.

Source: US Flow of Funds and Barclays Capital. See online Data Appendix for details.

constructed above. In the model we will distinguish between leverage as is observed in the data (i.e., taking only partially into account changes in asset valuations) and fully marked-to-market leverage. This will allow us to investigate the cyclical properties of marked-to-market leverage vis-à-vis (the model counterpart of) observed leverage.

II. The Model

The model economy is composed of four types of agents: households, firms, institutional investors, and banks. On the financial side, the model structure is as follows. Households lend to institutional investors in the form of deposits and equity. Institutional investors use the latter funds to lend to banks in the form of short-term debt. Banks combine this external funding and their own accumulated net worth so as to provide funding for firms. We assume no frictions in the relationship between banks and firms, such that the Modigliani-Miller theorem applies to firm financing. For simplicity and without loss of generality, we follow Gertler and Karadi (2011) in assuming that firms issue perfectly state-contingent debt only, which can be interpreted as equity. Banks and firms are segmented across a continuum of islands. Firms are hit by island-specific shocks to effective capital.[24] Banks are thus

[24] In introducing idiosyncratic shocks to effective capital, we follow Bernanke, Gertler, and Gilchrist (1999). Christiano, Motto, and Rostagno (2003, 2014) allow for exogenous variation over time in the dispersion of such idiosyncratic shocks.

exposed to island-specific risk; each period, a fraction of them default on their debt and declare bankruptcy. Bank debt is not guaranteed and is therefore risky. As in Adrian and Shin (2014), moral hazard (of the risk-shifting type) creates friction in the flow of funds from institutional investors to banks. Institutional investors operate economy-wide and diversify perfectly across islands; in fact, their only role in our model is to insulate households from island-specific risk, which allows us to make use of the representative household construct.

The real side of the model is fairly standard. At the end of each period, after production has taken place, firms use bank funding to purchase physical capital from households. At the beginning of the following period, firms combine their stock of effective capital and households' supply of labor to produce a final good. The latter is purchased by households for consumption and investment purposes. After production, firms sell their depreciated capital to households, who use the latter and final goods to produce new capital. The markets for labor, physical capital, and the final good are all economy-wide.

We now analyze the behavior of each type of agent. All variables are expressed in real terms, with the final good acting as the numeraire.

A. *Households*

The representative household's utility is $E_0 \sum_{t=0}^{\infty} \beta^t [u(C_t) - v(L_t)]$, where C_t is consumption and L_t is labor supply. In addition to consuming and supplying labor, households save in the form of deposits (D_t) and equity (N_t^{II}) of institutional investors. They also build new capital goods K_{t+1} using the technology $K_{t+1} = [1 - S(I_t/I_{t-1})] I_t + (1-\delta)\xi_t K_t$, where I_t are final goods used for investment purposes, and $(1-\delta)\xi_t K_t$ is depreciated effective capital purchased from firms after production in period t, where δ is the depreciation rate and ξ_t is an aggregate shock to the quality of capital. The function S satisfies $S(1) = S'(1) = 0$ and $S''(1) \equiv \zeta > 0$. The budget constraint is

$$\begin{aligned} C_t + I_t + N_t^{II} + D_t + Q_t(1-\delta)\xi_t K_t \\ = W_t L_t + R_t^N N_{t-1}^{II} + R_{t-1}^D D_{t-1} + Q_t K_{t+1} + \Pi_t^b, \end{aligned}$$

where R_{t-1}^D is the riskless gross deposit rate, R_t^N is the gross return on institutional investor equity, W_t is the wage, Q_t is the price of capital goods, and Π_t^b are lump-sum dividend payments (net of equity injections) from the household's ownership of banks. The first order conditions are

$$(2) \qquad 1 = E_t[\Lambda_{t,t+1} R_t^D], \quad 1 = E_t[\Lambda_{t,t+1} R_{t+1}^N], \quad W_t = \frac{v'(L_t)}{u'(C_t)},$$

$$1 = Q_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + E_t\left[\Lambda_{t,t+1} Q_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right],$$

where $\Lambda_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$ is the stochastic discount factor.

B. *Firms*

The final good is produced by perfectly competitive firms. As in Kiyotaki and Moore (2008), we assume that firms are segmented across a continuum of "islands," indexed by $j \in [0, 1]$.

In each island there are two types of firms, which we will refer to as "standard" and "substandard" firms. Each firm's type follows an i.i.d. process over time, with probability one-half each; the type for time t is drawn at the end of $t-1$, after production has taken place. Both types differ in the following dimension. At the start of each period, in addition to the common capital quality shock ξ_t, firms receive an *island-specific* shock to effective capital. Let $\omega_t^j \geq 0$ or $\tilde{\omega}_t^j \geq 0$ denote the shock received by standard and substandard firms, respectively, in island j. Letting K_t^j denote the beginning-of-period capital stock of standard firms in island j, the combined shock $\omega_t^j \xi_t$ changes their amount of effective capital to $\omega_t^j \xi_t K_t^j$, and analogously for substandard firms. Both ω_t^j and $\tilde{\omega}_t^j$ are i.i.d. over time and across islands, and independent of each other. Let $F(\omega; \sigma_{t-1}) \equiv F_{t-1}(\omega)$ and $\tilde{F}(\tilde{\omega}; \sigma_{t-1}) \equiv \tilde{F}_{t-1}(\tilde{\omega})$ denote the cumulative distribution functions of time-t island-specific shocks received by standard and substandard firms, respectively. The term σ_{t-1} is an exogenous process that controls the time-t dispersion of both distributions; notice that the dispersion indicator is known one period in advance. The mean of both distributions is assumed to be time-invariant, and that of the standard technology is normalized to one: $\int \omega \, dF_t(\omega) \equiv E(\omega) = 1$, $\int \tilde{\omega} \, d\tilde{F}_t(\tilde{\omega}) \equiv E(\tilde{\omega})$.

The role of the two-types assumption will be made explicit in Section IID, when we discuss the banking sector. As of now, it suffices to note that firms which draw the substandard type in a given period do not operate in equilibrium. Therefore, in what follows our notation refers to standard-type firms only, unless otherwise indicated.

At the beginning of period t, effective capital is combined with labor to produce units of final good, Y_t^j, according to a Cobb-Douglas technology,

$$Y_t^j = Z_t \left(\omega_t^j \xi_t K_t^j\right)^{\alpha} \left(L_t^j\right)^{1-\alpha}, \tag{3}$$

where Z_t is an exogenous aggregate TFP process. The firm maximizes operating profits, $Y_t^j - W_t L_t^j$, subject to (3). The first order condition with respect to labor implies that the effective capital-labor ratio is equalized across islands:

$$\frac{\omega_t^j \xi_t K_t^j}{L_t^j} = \left(\frac{W_t}{(1-\alpha) Z_t}\right)^{1/\alpha}, \tag{4}$$

for all j. The firm's profits then equal $Y_t^j - W_t L_t^j = R_t^k \omega_t^j \xi_t K_t^j$, where $R_t^k \equiv \alpha Z_t \left[(1-\alpha) Z_t / W_t\right]^{(1-\alpha)/\alpha}$ is the common return on effective capital. After production, the firm sells the depreciated effective capital $(1-\delta)\omega^j \xi_t K_t^j$ to households at price Q_t. The total cash flow from the firm's investment project equals the sum of operating profits and proceeds from the sale of depreciated capital:

$$R_t^k \omega^j \xi_t K_t^j + (1-\delta) Q_t \omega^j \xi_t K_t^j = \left[R_t^k + (1-\delta) Q_t\right] \omega^j \xi_t K_t^j. \tag{5}$$

Previously, at the end of period $t-1$, the firm bought K_t^j units of new capital at price Q_{t-1} for production in t.[25] In order to finance this purchase, the firm issued a number A_{t-1}^j of claims on the period-t cash flow, equal to the number of capital units acquired. The firm's balance sheet constraint at the end of period $t-1$ is thus $Q_{t-1}K_t^j = Q_{t-1}A_{t-1}^j$. Since the capital purchase is financed entirely by state-contingent debt, the cash flow in (5) is paid off entirely to the lending banks.

C. *Banks*

In each island there exists a representative bank. Only the bank on island j has the technology to obtain perfect information about firms on that island (including their type: standard or substandard), monitor them, and enforce their contractual obligations.[26] This effectively precludes firms from obtaining funding from other sources, including households or institutional investors. As indicated before, banks finance firms in the form of perfectly state-contingent debt. After production in period t, island j's firm pays the bank the entire cash flow from the investment project, $\left[R_t^k + (1-\delta)\,Q_t\right]\xi_t\omega_t^j A_{t-1}^j$. The gross return on the bank's assets $Q_{t-1}A_{t-1}^j$ is thus the product of an aggregate component, $\frac{R_t^k + (1-\delta)\,Q_t}{Q_{t-1}}\,\xi_t \equiv R_t^A$, and an island-specific component, ω_t^j.

Regarding the liabilities side of its balance sheet, the bank borrows from institutional investors by means of one-period risky debt contracts.[27] Under the latter contract, at the end of period $t-1$ the bank borrows funds from the institutional investor in the amount B_{t-1}^j, and agrees to pay back a non-contingent amount $\overline{B}_{t-1}^j$ at the beginning of time t. At that point, the proceeds from the bank's assets, $R_t^A\omega_t^j Q_{t-1}A_{t-1}^j$, exceed the face value of its debt, $\overline{B}_{t-1}^j$, if and only if the island-specific shock ω_t^j exceeds a threshold level $\overline{\omega}_t^j$ given by

$$\overline{\omega}_t^j \equiv \frac{\overline{B}_{t-1}^j}{R_t^A\,Q_{t-1}A_{t-1}^j}. \tag{6}$$

The default threshold equals the face value of debt normalized by the bank's assets times their aggregate return. If $\omega_t^j \geq \overline{\omega}_t^j$ the bank honors its debt. If $\omega^j < \overline{\omega}_t^j$, the bank defaults, at which point the institutional investor seizes the bank's assets and cashes the resulting proceeds, $R_t^A\omega^j Q_{t-1}A_{t-1}^j$. The defaulting bank is then closed down. Notice that the default threshold $\overline{\omega}_t^j$ depends on R_t^A and is thus contingent on the aggregate state.

For non-defaulting banks, we assume that a random fraction $1-\theta$ of them close down for exogenous reasons each period, at which point the net worth accumulated

[25] The assumption that firms purchase (or repurchase) their entire capital stock each period is standard in the macro-finance literature (see, e.g., Bernanke, Gertler, and Gilchrist 1999; Gertler and Karadi 2011; Christiano, Motto, and Rostagno 2014). As explained by Bernanke, Gertler, and Gilchrist (1999), this modeling device ensures, realistically, that leverage restrictions or other financial constraints apply to the constrained borrowers (in this case, the banks) as a whole, not just to the marginal investment.

[26] The costs of these activities for the bank are assumed to be negligible.

[27] Following Adrian and Shin (2014), we restrict our attention to standard debt contracts. The analysis of more general financial contracts is beyond the scope of this paper.

in each bank is reverted to the household.[28] The remaining fraction θ of banks continue operating. For the latter, the flow of dividends distributed to the household is given by

$$\Pi_t^j = R_t^A \omega_t^j Q_{t-1} A_{t-1}^j - \overline{B}_{t-1}^j - N_t^j, \tag{7}$$

where N_t^j is net worth after dividends have been paid. We assume that continuing banks cannot issue new equity.[29] This implies the existence of a non-negativity constraint on dividends, $\Pi_t^j \geq 0$, or equivalently

$$N_t^j \leq R_t^A \omega_t^j Q_{t-1} A_{t-1}^j - \overline{B}_{t-1}^j. \tag{8}$$

Once the bank has decided how much net worth to hold, it purchases claims on firm cash flows, A_t^j, subject to its balance sheet constraint, $Q_t A_t^j = N_t^j + B_t^j$.

When borrowing from the institutional investor, the bank faces two constraints. First, a standard *participation constraint* requires that the expected payoff to the institutional investor from lending to the bank exceeds the expected payoff from lending at the riskless rate R_t^D. The latter payoff is given by $E_t \Lambda_{t,t+1} R_t^D B_t^j = B_t^j = Q_t A_t^j - N_t^j$, where we have used the household's Euler equation and the bank's balance sheet constraint. Therefore, the participation constraint takes the form

$$E_t \Lambda_{t,t+1} \left\{ R_{t+1}^A Q_t A_t^j \int^{\overline{\omega}_{t+1}^j} \omega \, dF_t(\omega) + \overline{B}_t^j \left[1 - F_t\left(\overline{\omega}_{t+1}^j\right) \right] \right\} \geq Q_t A_t^j - N_t^j. \tag{9}$$

Second, we introduce a risk-shifting moral hazard problem on the part of the bank, in the spirit of Adrian and Shin (2014). We assume that, once the bank has received the funding, it may choose to invest in either of the two firm types (standard and substandard) within its island. As explained in Section IIB, both types differ in the distribution of island-specific shocks, F_t and $\tilde{F}_t$. We make the following assumptions.

ASSUMPTION 1: $\int \omega \, d\tilde{F}_t(\omega) < \int \omega \, dF_t(\omega)$.

ASSUMPTION 2: *There exists a* ω_t^* *such that* $F_t\left(\omega_t^*\right) = \tilde{F}_t\left(\omega_t^*\right)$ *and* $(F_t(\omega) - \tilde{F}_t(\omega)) \times \left(\omega - \omega_t^*\right) > 0$ *for all* $\omega > 0$.

Assumption 1 states that the substandard technology has lower mean return and is thus inefficient. Assumption 2 states that the distribution function F_t cuts $\tilde{F}_t$ precisely

[28] The assumption of exogenous exit for borrowers (in our case, banks) is standard in the macro-finance literature; see Bernanke, Gertler, and Gilchrist (1999), and more recently Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). As explained by Bernanke, Gertler, and Gilchrist (1999), this device is intended to preclude the possibility that the borrower will ultimately accumulate enough wealth to be fully self-financing. Indeed, as we show below, in equilibrium banks have no incentive to pay dividends.

[29] Such a constraint may be justified theoretically by the existence of agency or informational frictions in equity financing (see, e.g., Myers and Majluf 1984).

once from below, at a point denoted by ω_t^*.[30] As we will see, these assumptions, together with limited liability, imply that the bank may be tempted to invest in the substandard technology, despite its lower expected return. In order to induce the bank to invest in the standard firm type, the institutional investor imposes an *incentive compatibility* (IC) constraint. Let $V_{t+1}(\omega, A_t^j, \overline{B}_t^j)$ denote the value function at time $t+1$ of a non-defaulting, continuing bank, to be defined below. The IC constraint requires that the expected payoff of financing the standard firm exceeds that of financing the substandard one:

$$(10)\quad E_t \Lambda_{t,t+1} \int_{\overline{\omega}_{t+1}^j} \left\{ \theta V_{t+1}\left(\omega, A_t^j, \overline{B}_t^j\right) + (1-\theta)\left[R_{t+1}^A Q_t A_t^j \omega - \overline{B}_t^j\right] \right\} dF_t(\omega)$$

$$\geq E_t \Lambda_{t,t+1} \int_{\overline{\omega}_{t+1}^j} \left\{ \theta V_{t+1}\left(\omega, A_t^j, \overline{B}_t^j\right) + (1-\theta)\left[R_{t+1}^A Q_t A_t^j \omega - \overline{B}_t^j\right] \right\} d\tilde{F}_t(\omega).$$

To understand the bank's incentives to finance one firm type or another, notice that its expected net payoff (in a particular aggregate state at time $t+1$) from investing in standard firms can be expressed as $R_{t+1}^A Q_t A_t^j \int_{\overline{\omega}_{t+1}^j} (\omega - \overline{\omega}_{t+1}^j)\, dF_t(\omega)$. The integral represents the value of a *call option* on island-specific returns with strike price equal to the default threshold, $\overline{\omega}_{t+1}^j$, which in turn equals the normalized face value of debt, $\overline{B}_t^j/(R_{t+1}^A Q_t A_t^j)$.[31] Intuitively, limited liability implies that the bank enjoys the upside risk in asset returns over and above the face value of its debt, but does not bear the downside risk, which is transferred to the institutional investor. The value of the call option on island-specific risk may be expressed as

$$\int_{\overline{\omega}_{t+1}^j} (\omega - \overline{\omega}_{t+1}^j)\, dF_t(\omega) \;=\; E(\omega) + \pi_t\left(\overline{\omega}_{t+1}^j\right) - \overline{\omega}_{t+1}^j,$$

where

$$(11)\qquad \pi_t\left(\overline{\omega}_{t+1}^j\right) \;\equiv\; \pi\left(\overline{\omega}_{t+1}^j; \sigma_t\right) \;\equiv\; \int^{\overline{\omega}_{t+1}^j} \left(\overline{\omega}_{t+1}^j - \omega\right) dF_t(\omega)$$

is the value of the *put option* on island-specific returns with strike price $\overline{\omega}_{t+1}^j$.[32] Therefore, given the normalized face value of its debt, $\overline{\omega}_{t+1}^j$, the bank's expected net payoff increases with the mean island-specific return, $E(\omega)$, but also with the put option value $\pi_t(\overline{\omega}_{t+1}^j)$. The same reasoning holds for the substandard technology; for the latter, the put option value $\tilde{\pi}_t(\overline{\omega}_{t+1}^j)$ is defined analogously to (11), with $\tilde{F}_t$ replacing F_t. Under Assumptions 1 and 2, we obtain the following results.[33]

[30] It can be shown that both assumptions imply second-order stochastic dominance (SOSD) of F_t over $\tilde{F}_t$. The proof is available upon request.

[31] For a pioneering analysis of the payoff structure of defaultable debt claims, equity stakes, and their relationship to option derivatives, see Merton (1974).

[32] The relationship between the values of a European call option and a European put option is usually referred to as the "put-call parity."

[33] The proof is in online Appendix B.

LEMMA 1:

(*i*) *The put option value is higher for the substandard investment strategy:* $\tilde{\pi}_t(\overline{\omega}^j_{t+1}) > \pi_t(\overline{\omega}^j_{t+1})$ *for all* $\overline{\omega}^j_{t+1} > 0$.

(*ii*) *The difference in put option values* $\Delta\pi_t(\overline{\omega}^j_{t+1}) \equiv \tilde{\pi}_t(\overline{\omega}^j_{t+1}) - \pi_t(\overline{\omega}^j_{t+1})$ *cuts* $E(\omega) - E(\tilde{\omega})$ *precisely once from below.*

Part (i) of Lemma 1 implies that, when choosing between investment strategies, the bank trades off the *higher mean return* of investing in standard firms against the *lower put option value*. Part (ii) implies that the incentive to invest in substandard firms, as captured by the gain in put option value $\Delta\pi_t(\overline{\omega}^j_{t+1})$, *increases* initially with the normalized debt burden $\overline{\omega}^j_{t+1}$ and eventually surpasses the loss in mean return $E(\omega) - E(\tilde{\omega})$. This static reasoning contains the essence of the mechanism through which the bank may be tempted to finance substandard technologies, even though the actual IC constraint (equation (10)) is intrinsically dynamic due to the presence of the continuation value V_{t+1}.

We are ready to spell out the bank's maximization problem. Let $\overline{V}_t(N^j_t)$ denote the bank's value function after paying out dividends and at the time of borrowing from the institutional investor. We then have the following Bellman equations:

$$V_t\left(\omega^j_t, A^j_{t-1}, \overline{B}^j_{t-1}\right) = \max_{N^j_t}\left\{\Pi^j_t + \overline{V}_t\left(N^j_t\right)\right\},$$

subject to (7) and (8); and

$$\overline{V}_t\left(N^j_t\right)$$

$$= \max_{A^j_t, \overline{B}^j_t} E_t\Lambda_{t,t+1}\int_{\overline{\omega}^j_{t+1}}\left[\theta V_{t+1}\left(\omega, A^j_t, \overline{B}^j_t\right) + (1-\theta)\left(R^A_{t+1}Q_tA^j_t\omega - \overline{B}^j_t\right)\right] dF_t(\omega),$$

subject to (6), (9), and (10). Let $\overline{b}^j_t \equiv \overline{B}^j_t/(Q_tA^j_t)$ denote the face value of debt normalized by the bank's assets. This allows us to express the default threshold as $\overline{\omega}^j_t = \overline{b}^j_{t-1}/R^A_t$. Online Appendix C proves the following result.

PROPOSITION 1 (Solution to the Bank's Problem): *Assume the model parameters satisfy* $0 < \beta R^A - 1 < (1-\theta)\beta R^A \int_{\overline{\omega}}(\omega - \overline{\omega})\, dF(\omega)$, *where* R^A *and* $\overline{\omega}$ *are the steady-state values of* R^A_t *and* $\overline{\omega}^j_t$, *respectively. Then the equilibrium dynamics of bank j in a neighborhood of the deterministic steady state are characterized by the following features:*

(*i*) *The bank optimally retains all earnings,*

$$N^j_t = \left(\omega^j_t - \frac{\overline{b}_{t-1}}{R^A_t}\right) R^A_t Q_{t-1}A^j_{t-1}, \tag{12}$$

where $\bar{b}_{t-1}$ *is equalized across islands, such that* $\bar{\omega}_t^j = \bar{\omega}_t = \bar{b}_{t-1}/R_t^A < \omega_t^*$ *for all* j.

(*ii*) *The IC constraint holds with equality. In equilibrium, the latter can be expressed as*

$$(13)\quad 1 - E(\tilde{\omega}) = E_t\left\{\frac{\Lambda_{t,t+1}R_{t+1}^A(\theta\lambda_{t+1}+1-\theta)}{E_t\Lambda_{t,t+1}R_{t+1}^A(\theta\lambda_{t+1}+1-\theta)}\left[\tilde{\pi}\left(\frac{\bar{b}_t}{R_{t+1}^A};\sigma_t\right)-\pi\left(\frac{\bar{b}_t}{R_{t+1}^A};\sigma_t\right)\right]\right\},$$

where λ_{t+1} *is the Lagrange multiplier associated to the participation constraint, which is equalized across islands.*

(*iii*) *The participation constraint holds with equality:*

$$(14)\qquad Q_tA_t^j = \frac{1}{1-E_t\Lambda_{t,t+1}R_{t+1}^A\left[\bar{\omega}_{t+1}-\pi(\bar{\omega}_{t+1};\sigma_t)\right]}N_t^j \equiv \phi_tN_t^j.$$

According to (13), the (normalized) debt repayment $\bar{b}_t$ is set such that the gain in mean return from investing in standard firms exactly compensates the bank for the loss in put option value. According to (14), the bank's demand for assets equals its net worth times a *leverage ratio* ϕ_t which is equalized across islands. Notice that leverage decreases with the left tail risk of the bank's portfolio, as captured by the put option value $\pi(\bar{\omega}_{t+1};\sigma_t)$. Intuitively, since all the downside risk in the bank's assets is borne by the institutional investor, a higher perception of such risk leads the latter to impose a tighter leverage constraint.

Once $\bar{b}_t$ and ϕ_t have been determined, it is straightforward to obtain the actual loan size, $B_t^j = (\phi_t - 1)N_t^j$, and its face value, $\bar{B}_t^j = \bar{b}_tQ_tA_t^j = \bar{b}_t\phi_tN_t^j$. The gross interest rate then equals $\bar{B}_t^j/B_t^j = \bar{b}_t\phi_t/(\phi_t-1)$.

D. *Institutional Investors*

A representative, perfectly competitive institutional investor collects funds from households in the form of deposits and equity, and lends these funds to banks through short-term risky debt. Its balance sheet is thus $N_t^{II} + D_t = B_t$, where $B_t = \int_0^1 B_t^j\,dj$. There is no friction in the relationship between households and institutional investors. We assume that equity is sufficiently high to absorb aggregate risk and thus make deposits effectively safe.[34] The institutional investor operates economy-wide and hence perfectly diversifies its portfolio across islands.

[34] The relative shares of equity and deposits are undetermined in equilibrium. A way to guarantee that deposits are risk-free is to introduce a lump-sum tax on households that covers the potential losses in the deposits.

The institutional investor's return from financing the island-j bank is $\min\{R_t^A \omega_t^j Q_{t-1} A_{t-1}^j, \overline{B}_{t-1}^j\} = R_t^A Q_{t-1} A_{t-1}^j \min\left\{\omega_t^j, \frac{\overline{b}_{t-1}}{R_t^A}\right\} = R_t^A \phi_{t-1} N_{t-1}^j \min\{\omega_t^j, \overline{\omega}_t\}$. Aggregating across islands and subtracting gross interest payments on deposits, we obtain the return on the institutional investor's equity,

$$\begin{aligned} R_t^N N_{t-1}^{II} &= R_t^A \phi_{t-1} \int_0^1 N_{t-1}^j \min\{\omega_t^j, \overline{\omega}_t\}\, dj - R_{t-1}^D D_{t-1} \\ &= R_t^A \phi_{t-1} N_{t-1} \left\{ \left[1 - F_{t-1}(\overline{\omega}_t)\right] \overline{\omega}_t + \int^{\overline{\omega}_t} \omega\, dF_{t-1}(\omega) \right\} - R_{t-1}^D D_{t-1}, \end{aligned}$$

where in the second equality we have used the fact ω^j is distributed independently from N_{t-1}^j, and where $N_{t-1} \equiv \int_0^1 N_{t-1}^j\, dj$ is aggregate bank net worth. The institutional investor distributes all earnings to the household in every period.

E. *Aggregation and Market Clearing*

Aggregate net worth of banks at the *end* of period t, N_t, is the sum of the net worth of both continuing and new banks: $N_t = N_t^{cont} + N_t^{new}$. From (12) and $Q_{t-1} A_{t-1}^j = \phi_{t-1} N_{t-1}^j$, we have that $N_t^j = R_t^A(\omega_t^j - \overline{\omega}_t)\, \phi_{t-1} N_{t-1}^j$. Aggregating across islands, we obtain the total net worth of continuing banks, $N_t^{cont} = \theta R_t^A \phi_{t-1} N_{t-1} \int_{\overline{\omega}_t}^{\cdot} (\omega - \overline{\omega}_t)\, dF_{t-1}(\omega)$, where we have used the fact that ω_t^j is distributed independently from N_{t-1}^j. Banks that default or exit the market exogenously are replaced by an equal number of new banks, $1 - \theta\left[1 - F_{t-1}(\overline{\omega}_t)\right]$. Each new bank is endowed by households with a fraction τ of the beginning-of-period value of total assets, $Q_t A_{t-1} \equiv \int_0^1 Q_t A_{t-1}^j\, dj$.[35] Therefore, $N_t^{new} = \left\{1 - \theta\left[1 - F_{t-1}(\overline{\omega}_t)\right]\right\} \tau Q_t A_{t-1}$. We thus have

$$\begin{aligned} N_t &= \theta R_t^A \phi_{t-1} N_{t-1} \int_{\overline{\omega}_t} (\omega - \overline{\omega}_t)\, dF_{t-1}(\omega) \\ &\quad + \left\{1 - \theta\left[1 - F_{t-1}(\overline{\omega}_t)\right]\right\} \tau Q_t A_{t-1}. \end{aligned} \tag{15}$$

New banks leverage their starting net worth with the same ratio as continuing banks. We thus have $Q_t A_t = \phi_t(N_t^{cont} + N_t^{new}) = \phi_t N_t$.

Aggregate net dividends to households from banks are given by $\Pi_t^b = (1 - \theta) R_t^A \phi_{t-1} N_{t-1} \int_{\overline{\omega}_t} (\omega - \overline{\omega}_t)\, dF_{t-1}(\omega) - N_t^{new}$. Market clearing for capital requires that total demand by firms equals total supply by households, $\int_0^1 K_t^j\, dj = K_t$. Total issuance of state-contingent claims by firms must equal total demand

[35] Our specification for equity injections into newly created banks follows Gertler and Karadi (2011).

by banks, $K_{t+1} = A_t$. Using (4) to solve for firm j's labor demand L_t^j, aggregating across islands and imposing labor market clearing, we have

$$\int_0^1 L_t^j \, dj = \left(\frac{(1-\alpha) Z_t}{W_t}\right)^{1/\alpha} \xi_t \int_0^1 \omega_t^j K_t^j \, dj = \left(\frac{(1-\alpha) Z_t}{W_t}\right)^{1/\alpha} \xi_t K_t = L_t, \tag{16}$$

where we have used the facts that ω_t^j and K_t^j are distributed independently and that ω_t^j has unit mean. Equations (4) and (16) then imply that $\omega^j K_t^j / L_t^j = K_t / L_t$. Using the latter and (3), aggregate supply of the final good equals $Y_t \equiv \int_0^1 Y_t^j \, dj = Z_t \left(\frac{L_t}{\xi_t K_t}\right)^{1-\alpha} \xi_t \int_0^1 \omega^j K_t^j \, dj = Z_t (\xi_t K_t)^\alpha L_t^{1-\alpha}$. Finally, total supply of the final good must equal consumption and investment demand by households, $Y_t = C_t + I_t$.

III. Model Properties

Having laid out our model in the previous section, it is now worthwhile to explore some of its theoretical properties and transmission mechanisms.

A. *Comparison to Real Business Cycle (RBC) Model*

Online Appendix D summarizes the equilibrium conditions in our model, and compares it with a standard RBC model. As we show there, the RBC model shares all its equilibrium conditions with our model, except for the investment Euler equation, given by

$$1 = E_t\left\{\Lambda_{t,t+1} R_{t+1}^A\right\}, \tag{17}$$

where $R_{t+1}^A = \frac{(1-\delta)\xi_{t+1} Q_{t+1} + \alpha Y_{t+1}/K_{t+1}}{Q_t}$. The equation that determines leverage in our framework (14) can be rewritten in an analogous form,

$$1 = E_t\left\{\Lambda_{t,t+1} R_{t+1}^A \left[\overline{\omega}_{t+1} - \pi(\overline{\omega}_{t+1}; \sigma_t)\right] \frac{\phi_t}{\phi_t - 1}\right\}. \tag{18}$$

A comparison of equations (18) and (17) reveals that the term $\left[\overline{\omega}_{t+1} - \pi(\overline{\omega}_{t+1}; \sigma_t)\right] \times \frac{\phi_t}{\phi_t - 1} \equiv \Theta_{t+1}(\sigma_t)$ in equation (18) is a sufficient statistic for measuring the difference in equilibrium dynamics between both models. Using $\phi_t = Q_t A_t / N_t$ and the definition of the put option value $\pi_t(\overline{\omega}_{t+1})$ in equation (11), we can write

$$R_{t+1}^A \Theta_{t+1}(\sigma_t) = R_{t+1}^A \left\{\int^{\overline{\omega}_{t+1}} \omega \, dF_t(\omega) + \overline{\omega}_{t+1}\left[1 - F_t(\overline{\omega}_{t+1})\right]\right\} \frac{Q_t A_t}{Q_t A_t - N_t} = \frac{R_{t+1}^A Q_t A_t \int^{\overline{\omega}_{t+1}} \omega \, dF_t(\omega) + \overline{B}_t\left[1 - F_t(\overline{\omega}_{t+1})\right]}{B_t}, \tag{19}$$

where in the second equality we have used $\overline{\omega}_{t+1} = \overline{B}_t/(R^A_{t+1} Q_t A_t)$ and $Q_t A_t - N_t = B_t$. Expression (19) is just the return on aggregate risky debt, B_t. Therefore, $\Theta_{t+1}(\sigma_t)$ captures the fraction of the total return on capital, R^A_{t+1}, that is received by the household (through the institutional investor). In the absence of financial frictions, households receive the entire return on capital and $\Theta_{t+1}(\sigma_t)$ is simply 1.[36] With financial frictions, we generally have $\Theta_{t+1}(\sigma_t) \neq 1$, which drives a wedge between investment decisions in our model and in the RBC model.

B. *The Volatility-Leverage Channel*

A recent financially oriented literature argues that the volatility of returns on the assets held by borrowers is a key determinant of their leverage. For example, Brunnermeier and Pedersen (2009) analyze how an increase in the volatility of asset prices leads investors to demand higher margins, thus forcing borrowers to deleverage. Similarly, Fostel and Geanakoplos (2008) and Geanakoplos (2010) consider shocks that not only decrease the expected asset returns but also their volatility; these shocks, which the authors refer to as "scary bad news," lead to tighter margins as lenders protect themselves against increased uncertainty. From a more macro perspective, recent work following the lead of Bloom (2009) suggests that exogenous changes in volatility are an important driving force behind business cycle fluctuations (see, e.g., Arellano, Bai, and Kehoe 2012; Bloom et al. 2012; Christiano, Motto, and Rostagno 2014; Gilchrist, Sim, and Zakrajšek 2010; Kiley and Sim 2011). Much of this literature is inspired by the 2007–2009 financial crisis, in which higher "uncertainty" or "risk" perception (concepts that are closely related to that of volatility) about the solvency of financial intermediaries is widely believed to have played an important role.

In the standard RBC model, the absence of the financial wedge $\Theta_{t+1}(\sigma_t)$ implies that shocks to cross-sectional volatility, σ_t, have no effect whatsoever in the latter model. In our framework, on the contrary, the presence of financial frictions opens a link between cross-sectional volatility in asset returns, bank leverage, and investment dynamics.

We now make two further assumptions on the distributions of island-specific shocks.

ASSUMPTION 3: *An increase in island-specific volatility increases the put option value of the standard technology:* $\frac{\partial \pi(\overline{\omega};\sigma)}{\partial \sigma} > 0$.

ASSUMPTION 4: *An increase in island-specific volatility increases the gain in put option value from the substandard technology:* $\frac{\partial \Delta\pi(\overline{\omega};\sigma)}{\partial \sigma} = \frac{\partial \tilde{\pi}(\overline{\omega};\sigma)}{\partial \sigma} - \frac{\partial \pi(\overline{\omega};\sigma)}{\partial \sigma} > 0$.

Both assumptions are relatively weak. They require that, following an increase in the dispersion of island-specific shocks, downside risk (as measured by the put option values π and $\tilde{\pi}$) goes up for both firm types, and that it does so by more for the substandard firms.

[36] Assuming no financial frictions is equivalent to assuming that households themselves own the capital and then rent it to firms, as is typically done in the RBC literature.

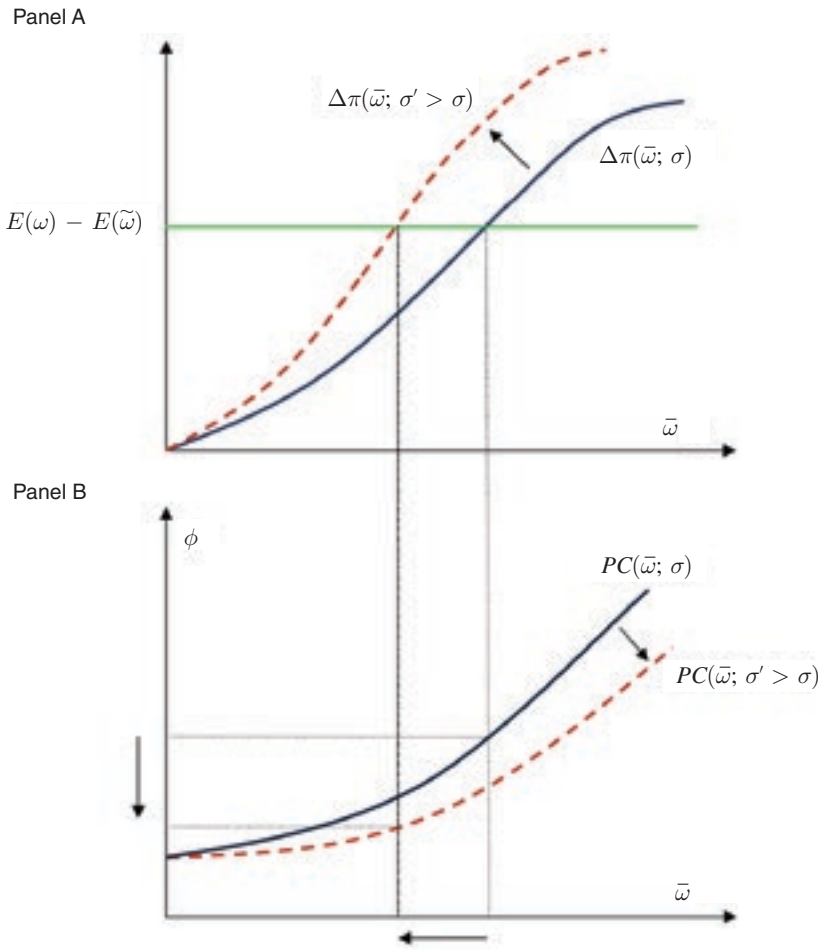

FIGURE 3. THE VOLATILITY-LEVERAGE CHANNEL

Under our distributional assumptions, an increase in the standard deviation of island-specific shocks, σ_t, induces a reduction in the leverage ratio of banks, via a mechanism sketched in Figure 3. The upper subplot represents the steady-state counterpart of the IC constraint (equation (13)). The upward-sloping line is the gain in left tail risk from investing in the substandard firm segment, $\Delta\pi(\overline{\omega};\, \sigma) = \tilde{\pi}(\overline{\omega};\, \sigma) - \pi(\overline{\omega};\, \sigma)$, which is an increasing function of the default threshold or (normalized) debt repayment, $\overline{\omega} = \overline{b}/R^A$.[37] The horizontal line is the loss in mean return, $E(\omega) - E(\tilde{\omega}) = 1 - \int \omega\, d\tilde{F}(\omega,\, \sigma)$. The IC constraint requires that the gain in left tail risk from investing in the substandard technology does not exceed the loss in mean return; since the constraint is binding in equilibrium, $\overline{\omega}$ is determined by the intersection of both lines. Consider now an increase in cross-sectional volatility, σ. Under Assumption 4, ceteris paribus the $\Delta\pi(\overline{\omega};\, \sigma)$ schedule rotates upwards and $\overline{\omega}$

[37] Remember that $\Delta\pi'(\overline{\omega};\, \sigma) = \tilde{F}(\overline{\omega};\, \sigma) - F(\overline{\omega};\, \sigma) > 0$, where the inequality follows from Assumption 2 and the fact that in equilibrium $\overline{\omega} < \omega^*$.

goes down. Intuitively, since higher volatility makes it more attractive for the bank to invest inefficiently, the institutional investor reduces the (normalized) face value of debt so as to discourage the bank from doing so.

The lower subplot of Figure 3 represents the steady-state counterpart of the participation constraint, $\phi = \left\{1 - \beta R^A\left[\overline{\omega} - \pi(\overline{\omega}; \sigma)\right]\right\}^{-1} \equiv PC(\overline{\omega}; \sigma)$, in $(\overline{\omega}, \phi)$ space. The latter is an upward-sloping relationship.[38] Ceteris paribus, the increase in σ has a double effect on leverage. First, the participation constraint schedule rotates down as a result of Assumption 3, which reduces leverage for a given $\overline{\omega}$. Intuitively, higher volatility of island-specific shocks increases the downside risk $\pi(\overline{\omega}; \sigma)$ of the bank's assets, which reduces the investor's expected payoff; in order to induce the investor to lend, the bank reduces its demand for funds as a fraction of its net worth. Second, the reduction in $\overline{\omega}$ through the IC constraint produces a leftward movement *along* the leverage schedule, thus further reducing leverage. Both effects are mutually reinforcing.[39]

IV. Quantitative Analysis

A. *Calibration, Steady State, and Measurement*

We calibrate our model to the US economy for the period 1981:I–2014:II. The parameters are shown in Table 2. We may divide the parameters between those that are standard in the RBC literature, and those that are particular to this model. From now onwards, we let variables without time subscripts denote steady-state values.

We set the RBC parameters to standard values. In particular, we set $\beta = 0.99 = 1/R$, $\alpha = 0.36 = 1 - WL/Y$, $\delta = 0.025 = I/K$, which are broadly consistent with long-run averages for the real interest rate, the labor share, and the investment to capital ratio. We target a capital-output ratio of $K/Y = 8$, which is consistent with a ratio of investment over GDP of 20 percent, roughly in line with the historical evidence. We then have $R^A = \alpha(Y/K) + 1 - \delta = 1.02$. Our functional forms for preferences are standard: $u(C) = \log(C)$, $v(L) = L^{1+\varphi}/(1+\varphi)$. We set $\varphi = 1$, in line with other macroeconomic studies (e.g., Comin and Gertler 2006). We assume $S(x) = \frac{1}{2}\zeta(x-1)^2$; in equilibrium, ζ is the inverse elasticity of I_t with respect to the price of capital, which we set to 0.5 as in Jermann and Quadrini (2012). We assume an AR(1) process for (log)TFP, $\log\left(Z_t/\overline{Z}\right) = \rho_z \log\left(Z_{t-1}/\overline{Z}\right) + \varepsilon_t^z$, where $\varepsilon_t^z \overset{iid}{\sim} N(0, \sigma_z)$. Our empirical counterpart for $\log\left(Z_t/\overline{Z}\right)$ is the Federal Reserve Bank of San Francisco-CSIP quarterly log TFP series, after being linearly detrended. We then choose ρ_z and σ_z so as to match their empirical counterparts.We choose $\overline{Z}$ such that steady-state output is normalized to one.

[38] The investor's expected payoff from lending to the bank equals the bank's assets times $\beta R^A\left[\overline{\omega} - \pi(\overline{\omega})\right]$. Since $\pi'(\overline{\omega}) = F(\overline{\omega}) < 1$, the latter payoff *increases* with $\overline{\omega}$. This allows the bank to borrow more as a fraction of its net worth (i.e., to increase its leverage) while still persuading the investor to lend the funds.

[39] It can be shown that $\frac{d\phi}{d\sigma} = \phi^2 \beta R^A\left\{\left[1 - F(\overline{\omega}; \cdot)\right]\frac{d\overline{\omega}}{d\sigma} - \frac{\partial \pi}{\partial \sigma}\right\} < 0$. That is, the reduction in leverage produced by a (marginal) increase in volatility is larger the more leveraged banks are in steady state (ϕ), the higher the return on assets (R^A), and the higher the fraction of surviving banks in each period $\left(1 - F(\overline{\omega}; \cdot)\right)$, for given reduction in the default threshold $\left(\frac{d\overline{\omega}}{d\sigma}\right)$ and increase in the put option value $\left(\frac{\partial \pi}{\partial \sigma}\right)$. The proof is available upon request.

Table 2—Model Parameters

Parameter	Value	Description	Source/target
Standard parameters			
β	0.99	discount factor	$R^4 = 1.04$
α	0.36	capital share	$WL/Y = 0.64$
δ	0.025	depreciation rate	$I/K = 0.025$
φ	1	inverse labor supply elasticity	macro literature
ζ	0.5	inverse investment elasticity	Jermann-Quadrini (2012)
$\overline{Z}$	0.5080	steady-state TFP	$Y = 1$
ρ_z	0.9369	autocorrelation TFP	FRBSF-CSIP TFP
σ_z	0.0066	standard deviation TFP	FRBSF-CSIP TFP
Nonstandard parameters			
σ	0.1313	steady-state island-spec. volatility	average leverage ($\phi = 4$)
η	1.0275	variance substandard technology	$(\overline{R}/R)^4 - 1 = 0.25\%$
ψ	0.0001	mean substandard technology	corr(I, ϕ)
τ	0.1599	equity injections new banks	$I/Y = 0.2$
θ	0.86	continuation prob. banks	SD(I)
ρ_σ	0.9831	autocorr. island-specific volatility	NBER-CES manuf. TFP
σ_σ	0.0171	SD island-specific volatility	NBER-CES manuf. TFP
ρ_ξ	0.3591	autocorr. capital quality	estimated with equity data
σ_ξ	0.0081	standard deviation capital quality	estimated with equity data

Regarding the nonstandard parameters in our model, our calibration strategy is as follows. Following Gertler and Karadi (2011), we target a steady-state leverage ratio ϕ of 4.[40] Krishnamurthy, Nagel, and Orlov (2014) document that the spread between key short-term liabilities issued by US intermediaries (such as repos backed by corporate debt and private-label ABS) and the Fed funds rate was close to zero in the pre-crisis period. Based on this, we target a spread in bank debt of 25 annualized basis points. The gross interest rate then equals $\overline{R} = R(1.0025)^{1/4}$. The face value of bank debt (normalized by assets) is then $\overline{b} = \overline{R}(\phi - 1)/\phi = 0.76$. This implies a default threshold of $\overline{\omega} = \overline{b}/R^A = 0.74$.

Island-specific shocks are assumed to be lognormally distributed, both for the standard and the substandard firm segment:

$$\log \omega \overset{iid}{\sim} N\left(\frac{-\sigma_t^2}{2}, \sigma_t\right), \quad \log \tilde{\omega} \overset{iid}{\sim} N\left(\frac{-\eta\sigma_t^2 - \psi}{2}, \sqrt{\eta}\,\sigma_t\right),$$

for $\psi > 0$ and $\eta > 1$. Therefore, $F(\omega; \sigma_t) = \Phi\left(\frac{\log(\omega) + \sigma_t^2/2}{\sigma_t}\right)$, where $\Phi(\cdot)$ is the standard normal cdf; and analogously for $\tilde{F}$. These functional forms are consistent with our assumptions in the model section. First, we have $E(\tilde{\omega}) = e^{-\psi/2} < 1 = E(\omega)$, i.e., both means are time-invariant (such that an

[40] Our model, like Gertler and Karadi's (2011), implies that banks and firms are essentially a single sector. As explained by Gertler and Karadi (2011), a steady-state leverage ratio of 4 is a reasonable compromise between the high leverage ratios of financial intermediaries (typically in the range of 15 to 30), and that of corporate and non-corporate businesses (closer to 2 in the aggregate). Our results are however robust to choosing ϕ based on intermediary leverage ratios.

increase in σ_t constitutes a mean-preserving spread) and the standard technology has a higher expected payoff. Also, the fact that $\eta > 1$ (i.e., the substandard technology has a higher variance) guarantees that F_t cuts $\tilde{F}_t$ once from below.[41] We have the following expressions for the values of the unit put options on island-specific risk:[42]

$$(20) \quad \pi(\overline{\omega}_t; \sigma_{t-1}) = \overline{\omega}_t \Phi\left(\frac{\log(\overline{\omega}_t) + \sigma_{t-1}^2/2}{\sigma_{t-1}}\right) - \Phi\left(\frac{\log(\overline{\omega}_t) - \sigma_{t-1}^2/2}{\sigma_{t-1}}\right),$$

$$(21) \quad \tilde{\pi}(\overline{\omega}_t; \sigma_{t-1}) = \overline{\omega}_t \Phi\left(\frac{\log(\overline{\omega}_t) + \frac{\psi + \eta\sigma_{t-1}^2}{2}}{\sqrt{\eta}\,\sigma_{t-1}}\right) - e^{-\psi/2}\Phi\left(\frac{\log(\overline{\omega}_t) + \frac{\psi - \eta\sigma_{t-1}^2}{2}}{\sqrt{\eta}\,\sigma_{t-1}}\right).$$

The standard deviation of island-specific shocks to standard firms is assumed to follow an AR(1) process in logs, $\log(\sigma_t/\sigma) = \rho_\sigma \log(\sigma_{t-1}/\sigma) + \varepsilon_t^\sigma$, where $\varepsilon_t^\sigma \overset{iid}{\sim} N(0, \sigma_\sigma)$. In order to calibrate σ, we notice that the participation constraint (equation (14)) in the steady state implies $\pi(\overline{\omega}; \sigma) = \overline{\omega} - (1 - 1/\phi)/\beta R^A = 0.0005$. Using the steady-state counterpart of (20), we can then solve for $\sigma = 0.1313$. In order to calibrate the parameters governing the dynamics of island-specific volatility $(\rho_\sigma, \sigma_\sigma)$, we use data on industry-level TFP constructed by the National Bureau of Economic Research (NBER) and the US Census Bureau's Center for Economic Studies (CES).[43] We construct a time series for σ_t^2 by calculating the cross-sectional variance of the industry-level TFP series (in log deviations from a linear trend) at each point in time. Since the NBER-CES industry data are only available annually, we adapt Stock and Watson's (2010) method in order to distribute the annual dispersion series into a quarterly one.[44] Figure 4 displays the resulting series for σ_t, net of its sample mean. Fitting an autoregressive process to $\log(\sigma_t/\sigma)$, we obtain $\rho_\sigma = 0.9831$ and $\sigma_\sigma = 0.0171$.

Regarding the parameters of the substandard technology, ψ and η, we make use of the IC constraint in the steady state, $1 - e^{-\psi/2} = \tilde{\pi}(\overline{\omega}; \sigma) - \pi(\overline{\omega}; \sigma)$, where $\tilde{\pi}(\overline{\omega}; \sigma)$ is given by expression (21) in the steady state. We thus have one equation for two unknowns, ψ and η. We set ψ so as to match the correlation between investment and leverage, yielding $\psi = 0.0001$. We then use the IC constraint to solve for $\eta = 1.0275$.

The exogenous bank continuation rate θ and the bank equity injection parameter τ are calibrated as follows. In the steady state, the law of motion of bank net worth (equation (15)) becomes

$$(22) \qquad \frac{1}{\phi} = \theta R^A \int_{\overline{\omega}} (\omega - \overline{\omega})\, dF(\omega; \sigma) + \left\{1 - \theta\left[1 - F(\overline{\omega}; \sigma)\right]\right\} \tau,$$

[41] It can be shown that $F_t(\omega) = \tilde{F}_t(\omega)$ if and only if $\omega = \exp\left(\frac{\psi/\sqrt{\eta} + (\sqrt{\eta} - 1)\sigma_t^2}{2(1 - 1/\sqrt{\eta})}\right) \equiv \omega_t^* > 0$. It can also be shown that $F_t'(\omega_t^*)/\tilde{F}_t'(\omega_t^*) = \sqrt{\eta} > 1$. Since $F_t(\omega_t^*) = \tilde{F}_t(\omega_t^*)$ and $F_t'(\omega_t^*) > \tilde{F}_t'(\omega_t^*)$, it follows that F_t crosses $\tilde{F}_t$ once from below at $\omega = \omega_t^*$.

[42] The proof is available upon request.

[43] See Bloom (2009) for a study that uses the NBER-CES manufacturing industry database to construct a measure of time-varying industry-specific volatility.

[44] Stock and Watson's (2010) distribution method requires a higher frequency indicator. As a quarterly dispersion indicator, we use the cross-industry variance in quarterly industrial production, using data from the Federal Reserve Board's Industrial Production database. See online Data Appendix for details.

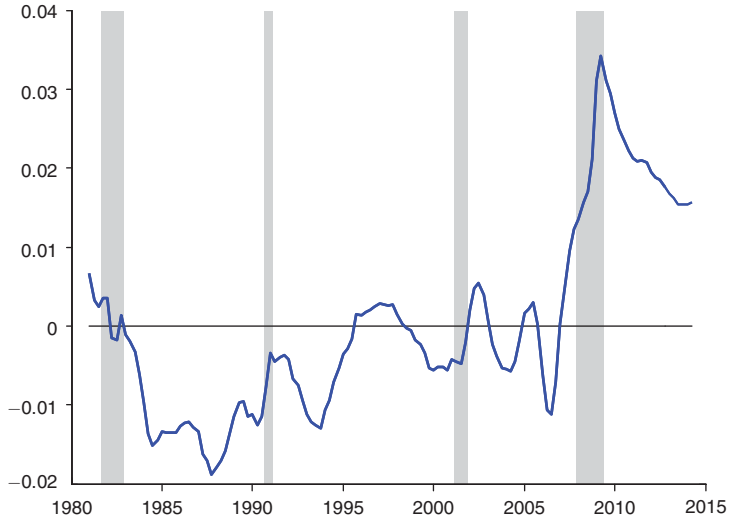

FIGURE 4. EMPIRICAL SERIES FOR THE CROSS-SECTIONAL VOLATILITY PROCESS

Notes: Demeaned series. Shaded areas represent NBER-dated recessions.

Source: NBER-CES industry database and FRB G17 files. See online Data Appendix for details.

where we have normalized by A. We set θ to 0.86, which allows us to broadly replicate the volatility of investment in the data. We then use (22) to solve for $\tau = 0.1599$.

Finally, (log)capital quality is assumed to follow an AR(1) process, $\log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + \varepsilon_t^\xi$, $\varepsilon_t^\xi \overset{iid}{\sim} N(0, \sigma_\xi)$. Unlike TFP and island-specific volatility, there is no convincing empirical proxy for capital quality. Instead, we use our calibrated model and the Kalman filter so as to estimate ρ_ξ and σ_ξ by maximum likelihood.[45] We use bank equity as an observable, together with the same TFP and volatility series that were used to calibrate both exogenous processes.[46]

As regards the observed bank equity series, three remarks are in order. First, we choose bank equity as observable because it is not our main focus of analysis, which is instead the cyclical behavior of bank leverage. Second, our model assumes a single leveraged intermediary (bank) sector. Thus, as an empirical counterpart for bank equity, we use the equity capital series for the "aggregate" intermediary sector that we constructed in the empirical section. Third, as discussed in that section, the aggregate intermediary sector that serves as empirical proxy for the bank sector in our model combines depository institutions, which do not fully mark their assets to market, with other subsectors that do mark their assets to market systematically, such as security broker/dealers and finance companies. In our model, the presence of asset prices (Q_t) allows us to distinguish between the MTM value of book equity, $Q_t A_t - B_t = N_t$, and book equity when the latter does not reflect asset price

[45] Our maximum likelihood estimates of ρ_ξ and σ_ξ in Table 2 are very precise, with standard deviations of 0.1059 (t-statistic 3.39) and 0.0006 (t-statistic 13.41), respectively. This makes it unnecessary to use priors and hence to perform Bayesian estimation.

[46] Notice that TFP and volatility need to be included as observables since both shocks affect the endogenous dynamics of bank equity.

variation, $A_t - B_t$. In order to make our model comparable to the data, we define (the model counterpart of) *observed* bank equity as

$$N_t^{obs} \equiv \omega^{MTM} N_t + \left(1 - \omega^{MTM}\right)\left(A_t - B_t\right) = (QA)_t^{obs} - B_t, \tag{23}$$

where $(QA)_t^{obs} \equiv \omega^{MTM} Q_t A_t + \left(1 - \omega^{MTM}\right) A_t$ is the *observed* value of bank assets, and ω^{MTM} is the fraction of assets that are marked-to-market. To calibrate the latter, we note that US-chartered depository institutions account for about two-thirds of total assets of the aggregate intermediary sector on average in our sample period. Consistently with our empirical analysis in Section I, we assume that up to three-fourths of all assets of depository institutions are *not* marked-to-market.[47] This leads us to a value of $1 - (2/3) \times (3/4) = 1/2$ for ω^{MTM}. We also define *observed* bank leverage:

$$\phi_t^{obs} \equiv \frac{(QA)_t^{obs}}{N_t^{obs}}. \tag{24}$$

Notice that if all assets were marked-to-market ($\omega^{MTM} = 1$), then observed leverage would simply equal the leverage ratio in our model, $\phi_t = Q_t A_t / N_t$, which we will henceforth refer to as *marked-to-market leverage*. From now on, we will compare the empirical fluctuations in leverage with those of ϕ_t^{obs}, while also reporting the cyclical properties of marked-to-market leverage ϕ_t.

B. *The Effects of TFP Shocks*

We follow the lead of the traditional RBC literature by exploring how well TFP shocks can explain the unconditional patterns found in the data. The column labeled "TFP" in Table 3 displays the moments of interests.[48] For comparison, we also show the empirical moments for our aggregate leveraged subsector.

Conditional on TFP shocks, the model fails dramatically at reproducing the volatility of leverage. It also fails to produce any procyclicality in the leverage ratio, either with respect to GDP or to assets.

To understand these results, we display the impulse-responses to a one-standard-deviation fall in TFP in Figure 5 (dashed-dotted line). The responses of the real aggregates are very similar to the ones that would be observed in the standard RBC model: output, investment, consumption, and labor all go down.[49] As regards the banking aggregates, the impact fall in asset prices produces a fall in equity proportionally larger than that of assets, and hence a counter-cyclical increase in leverage. Following the impact period, however, the leverage ratio is essentially back to

[47] For the remaining one-third (security broker/dealers and finance companies) we assume that assets are fully marked-to-market. As emphasized by Adrian and Shin (2014), book equity is fully marked-to-market for financial intermediaries that hold primarily marketable securities.

[48] We solve our model linearly around its steady state.

[49] The *expected* investment wedge Θ_{t+1} (the one that matters for investment) falls slightly on impact, implying a slightly larger drop in investment than would be the case in the RBC model.

Table 3—Business Cycle Statistics: Data and Model

	Data	Model				RBC
		TFP	Cap. quality	Volatility	All shocks	
Standard deviations (%)						
GDP	4.36	2.76	1.33	3.47	4.66	3.00
Consumption	3.11	2.16	2.29	2.66	4.17	3.17
Investment	14.11	6.26	3.63	12.68	14.89	7.71
Hours	5.49	0.50	0.56	1.27	1.51	0.83
Leverage (obs.)	10.00	0.74	2.90	8.42	9.26	—
Leverage (MTM)	—	1.98	4.94	9.41	11.10	—
Assets (obs.)	10.57	2.71	4.42	8.42	10.16	—
Assets (MTM)	—	2.81	4.64	8.59	10.43	—
Equity (obs.)	5.27	2.90	6.21	4.89	8.56	—
Equity (MTM)	—	3.89	8.24	7.69	12.05	—
Correlations with GDP						
Consumption	0.91	0.95	0.94	0.69	0.77	0.86
Investment	0.87	0.90	−0.55	0.79	0.70	0.53
Hours	0.91	0.72	−0.74	0.65	0.48	0.17
Leverage (obs.)	0.70	−0.11	−0.38	0.91	0.61	—
Leverage (MTM)	—	−0.24	−0.30	0.79	0.47	—
Assets (obs.)	0.62	0.83	0.99	0.98	0.89	—
Assets (MTM)	—	0.84	0.97	0.97	0.88	—
Equity (obs.)	−0.08	0.80	0.89	0.13	0.40	—
Equity (MTM)	—	0.73	0.73	0.11	0.34	—
Volatility	−0.48	0	0	−0.94	−0.71	—
Correlations with leverage (obs.)						
Investment	0.67	−0.09	−0.44	0.85	0.67	—
Assets (obs.)	0.87	−0.13	−0.42	0.83	0.61	—
Equity (obs.)	−0.15	−0.38	−0.76	−0.29	−0.35	—
Volatility	−0.40	−0.11	−0.07	−0.98	−0.93	—
Asset prices	−0.45	−0.91	−0.94	−0.19	−0.32	—

Notes: The model is solved linearly around its steady state. The data sample is 1981:I:2014:II. See online Data Appendix for details. "Obs." stands for the model counterparts for observed equity, assets, and leverage (see equations (23) and (24)); "MTM" stands for "marked-to-market."

its steady state. Intuitively, TFP shocks barely affect banks' incentives to invest in the substandard as opposed to the standard firm segment, and have thus little effect on the leverage constraint imposed by investors. Since bank leverage remains stable after the impact period, bank assets basically reproduce the response of bank net worth; that is, the effects of TFP shocks on bank credit operate mainly through the bank equity channel.

We now investigate to what extent TFP shocks can account for the actual evolution of the main real and financial aggregates of interest in our sample period. Figure 6 compares the historical series for GDP, investment, bank assets, and bank leverage with those generated by the model conditional on the observed TFP series.[50] TFP shocks account to a large extent for the fall in GDP and investment that took place in the 1981–1982 and 2008–2009 recessions. However, they fail completely at explaining the fluctuations in bank leverage or assets, such as their drastic fall in the last recession.

[50] Model-generated series in Figure 6 are Kalman-smoothed series conditional on the observed TFP series.

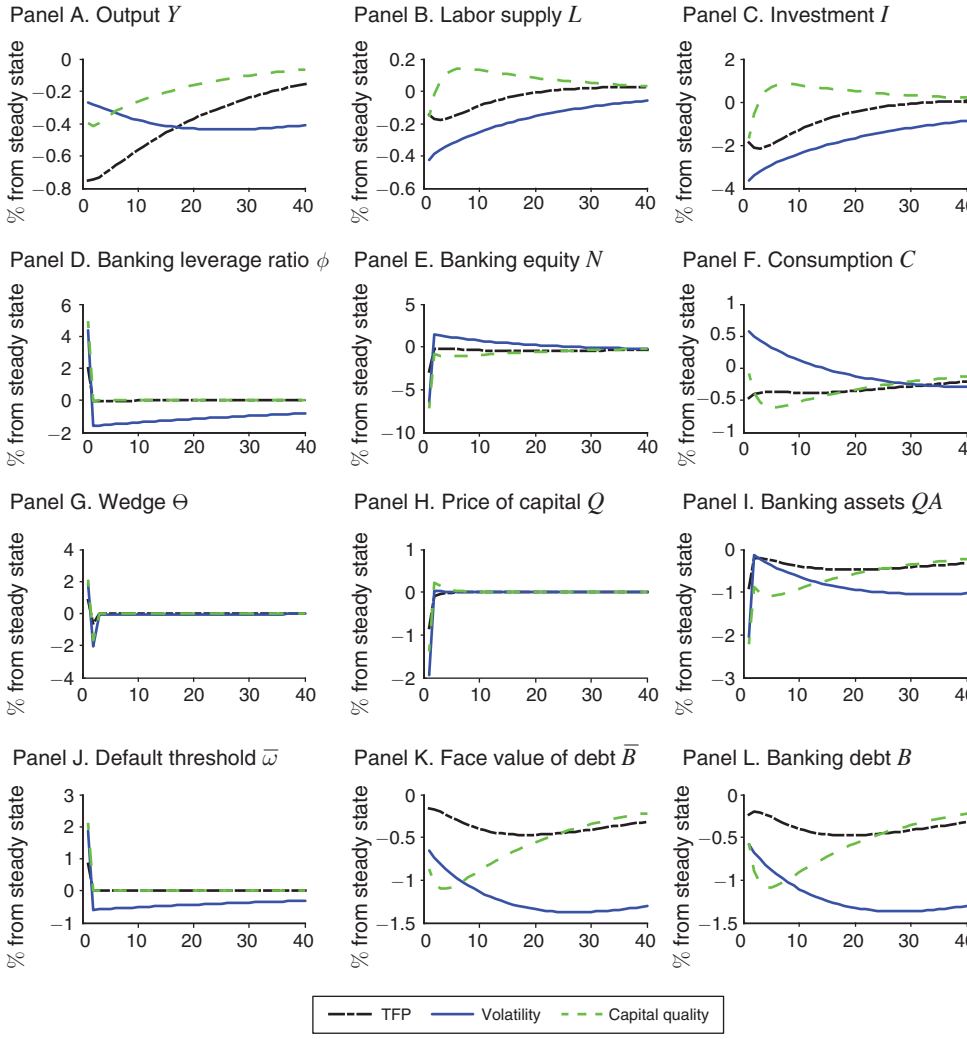

FIGURE 5. IMPULSE RESPONSES: TFP, CAPITAL QUALITY, AND VOLATILITY SHOCKS

Note: Leverage, assets, and equity are marked-to-market.

C. *The Effects of Capital Quality Shocks*

The column labeled "Cap. quality" in Table 3 displays business cycle statistics conditional on capital quality shocks. The latter generates fluctuations in leverage, both observed and marked-to-market, that are even more *counter*cyclical (with respect to GDP and assets) than those produced by TFP shocks. As illustrated in Figure 5 (dashed lines), a negative capital quality shock produces a larger drop in asset prices than that produced by a negative TFP shock, a logical consequence of the former's direct effect on the effective capital stock. As a result, it also produces a larger countercyclical increase in bank leverage.

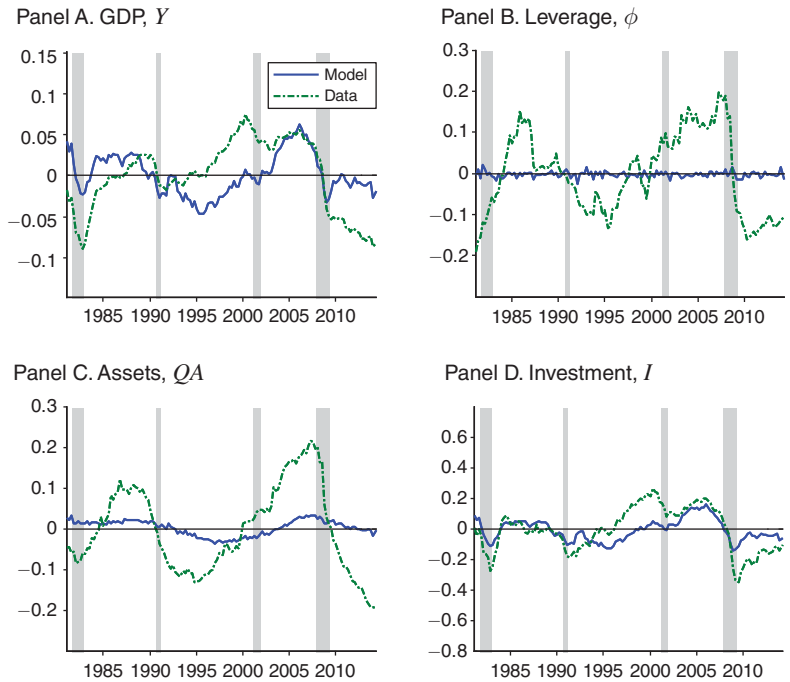

FIGURE 6. HISTORICAL AND MODEL-GENERATED SERIES CONDITIONAL ON TFP SHOCKS

Notes: All series as log deviations from trend (data) or steady state (model). Shaded areas represent NBER-dated recessions. Model leverage and assets are model counterparts of the observed series $(\phi_t^{obs}, (QA)_t^{obs})$.

Source: US Flow of Funds and Bureau of Economic Analysis. See online Data Appendix for details.

Notice that, as in the case of the TFP shock, bank leverage basically returns to its initial level right after the impact period. This is mostly due to the rapid post-impact increase in bank equity displayed in Figure 5. For both shocks, such recapitalization is explained by the quick recovery in the total return on assets and in the value of the call option on cross-sectional risk enjoyed by banks thanks to limited liability.[51] Both effects in turn are mostly determined by the quick post-impact recovery in asset prices.[52] The latter is best understood by analyzing the behavior of the expected financial wedge Θ_{t+1}, which as explained in Section IIIA is a key element of the investment demand schedule. As shown in Figure 5, on impact the *expected* wedge falls sharply and then returns quickly to steady state; the large shifts (first downward, then upward) that this produces ceteris paribus in the investment demand curve are the main factors responsible for the sharp impact fall and the equally sharp recovery in asset prices and bank equity.

[51] That is, the terms $R_t^A \phi_{t-1} N_{t-1}$ and $\int_{\overline{\omega}_t} (\omega - \overline{\omega}_t)\, dF_{t-1}(\omega)$ in equation (15) are the main determinants of the rapid recovery in N_t. By contrast, the equity injected into new banks (N_t^{new}) contributes relatively little to the equity responses, due mainly to its small weight in total equity $(N_{ss}^{new}/N_{ss} = 0.097$ under our calibration).

[52] Notice that $\frac{d}{d\overline{\omega}} \left(\int_{\overline{\omega}} (\omega - \overline{\omega})\, dF(\omega) \right) = F(\overline{\omega}) - 1 < 0$, i.e., the call option value is a decreasing function of the default threshold, $\overline{\omega}_t = \overline{b}_{t-1}/R_t^A$. Therefore, the sharp recovery in asset prices favors banks' post-impact recapitalization both by pushing up the aggregate return on assets, R_t^A, and by lowering the bank default threshold (see the lower left panel in Figure 5).

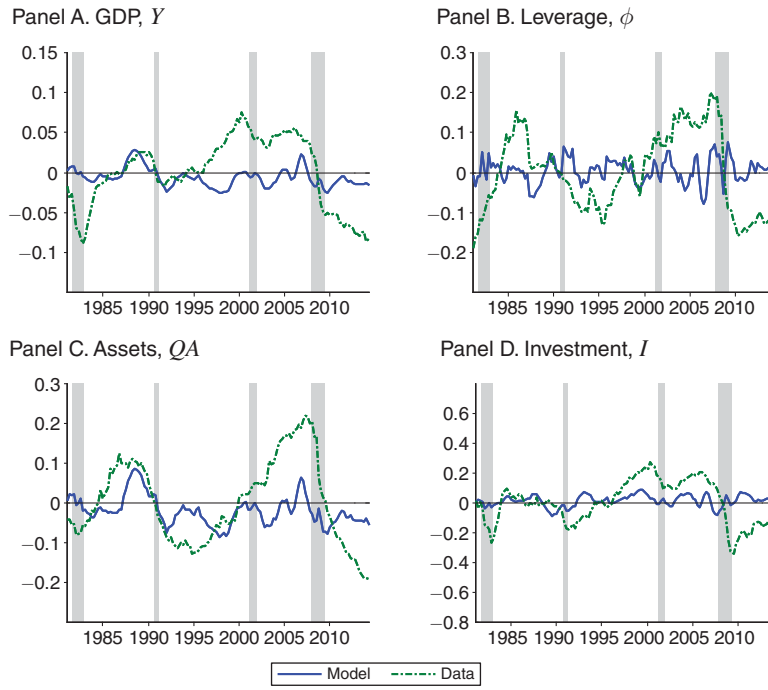

FIGURE 7. HISTORICAL AND MODEL-GENERATED SERIES CONDITIONAL ON CAPITAL QUALITY SHOCKS

Notes: All series as log deviations from trend (data) or steady state (model). Shaded areas represent NBER-dated recessions. Model leverage and assets are model counterparts of the observed series $(\phi_t^{obs}, (QA)_t^{obs})$.

Source: US Flow of Funds and Bureau of Economic Analysis. See online Data Appendix for details.

Figure 7 displays the model-generated series conditional on the smoothed series for capital quality shocks.[53] Unlike TFP shocks, capital quality shocks go some way towards explaining the fall in bank assets during the last recession.[54] Like TFP shocks, however, capital quality shocks are not able to explain the large and long-lasting fall in bank leverage that took place from 2008 onwards.

D. *The Effects of Volatility Shocks*

In Section III, we analyzed the volatility-leverage channel from a theoretical, partial equilibrium perspective. Here we assess such a mechanism from a quantitative, general equilibrium point of view. As shown by the column labeled "Volatility" in Table 3, volatility shocks generate large fluctuations in banks' leverage ratio, comparable to those in the data. Moreover, volatility shocks produce a strong positive comovement between leverage, on the one hand, and GDP and assets, on the other. This is true both for observed and marked-to-market leverage.

[53] Model-generated series in Figure 7 are Kalman-smoothed series conditional on the capital quality series obtained in Section IVA.

[54] As shown in Figure 5, capital quality shocks produce larger effects on the value of bank assets than TFP shocks.

The solid line in Figure 5 displays the responses to a one standard deviation increase in cross-sectional volatility. As in the case of the two first-moment shocks (TFP and capital quality), the volatility shock depresses asset prices and raises marked-to-market leverage on impact. However, as asset prices return to their steady state after the impact period, the leverage ratio *falls* below its steady state due to the volatility-channel explained in Section III.

As was the case for the first-moment shocks, bank equity experiences a rapid recovery after the impact period, in fact rising above steady state. The post-impact improvement in equity is largely due to the reduction in the default threshold below steady state, which in turn is due to the persistent reduction in banks' indebtedness produced by the volatility-leverage channel (see Figure 5).[55]

Overall, the drop in leverage dominates the increase in equity, thus leading to a reduction in bank assets. This produces a contraction in the capital stock, investment, and aggregate output.[56]

Figure 8 shows the model-generated series conditional on the observed volatility series (see Figure 4).[57] The increase in volatility that took place during the 2007–2009 financial crisis accounts for much of the fall in GDP during the 2008–2009 recession, as well as for the prolonged nature of that fall. Indeed, as we saw in Figure 5, volatility shocks have more persistent effects on GDP than either of the two first-moment shocks. The increase in volatility also accounts well for the large fall in investment and its subsequent recovery. Crucially, it also allows the model to reproduce the sharp fall in leverage, and much of the gradual and prolonged fall in bank assets.

As an additional test of the plausibility of the volatility-leverage channel, we compare the impulse responses to a volatility shock produced by the model with those in the data. With this purpose, we run the following VAR:

$$X_t = A_1 X_{t-1} + A_2 X_{t-2} + \varepsilon_t,$$

where the vector of observables X_t includes volatility, bank leverage, output, and bank assets: $X_t = [\log(\sigma_t), \log(\phi_t^{obs}), \log(Y_t), \log(QA)_t^{obs}]'$, and the matrix of covariances is $\Omega = E[\varepsilon_t' \varepsilon_t]$. We restrict the VAR so that the process that describes the evolution of volatility is an AR(1) with parameters ρ_σ and σ_σ, independent of the other shocks. The data sample is again 1981:I to 2014:II. Results are displayed in Figure 9. The model replicates well the sign of the empirical responses of output, leverage, and assets following an increase in volatility. The magnitudes of the model responses are also mostly within the 95 percent confidence intervals of the empirical

[55] We also note that the lending rate $\overline{R}_t$ contributes relatively little to the responses of equity and leverage to the volatility shock, a feature that is shared with the two first-moment shocks. Results are available upon request.

[56] Notice that an increase in volatility produces initially a (countercyclical) increase in consumption. This could be reversed by assuming that household spending (e.g., in an extended model version with durable purchases) is also affected by financial frictions similar to those in our model; or by assuming that such frictions affect firms' working capital, which would strengthen the fall in output and, ceteris paribus, in consumption. Notice in any case that consumption is *pro*cyclical conditional on volatility shocks, as it is in the data (see Table 3); this is because, after some quarters, the consumption response becomes (persistently) procyclical and this dominates the initial countercyclical increase.

[57] Model-generated series in Figure 8 are Kalman-smoothed series conditional on the observed volatility series.

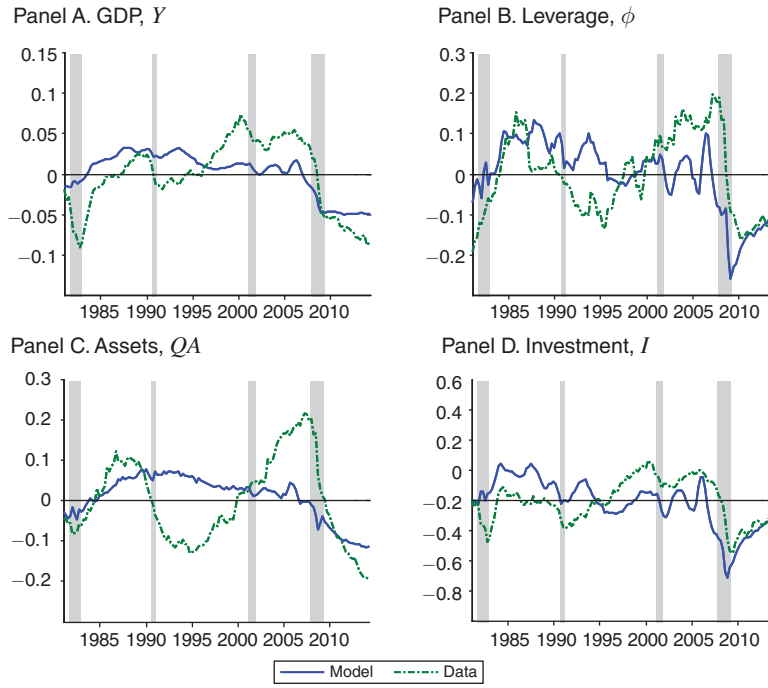

FIGURE 8. HISTORICAL AND MODEL-GENERATED SERIES CONDITIONAL ON VOLATILITY SHOCKS

Notes: All series as log deviations from trend (data) or steady state (model). Shaded areas represent NBER-dated recessions. Model leverage and assets are model counterparts of the observed series $(\phi_t^{obs}, (QA)_t^{obs})$.

Source: US Flow of Funds and Bureau of Economic Analysis. See online Data Appendix for details.

responses. We view these results as a clear success for the model, given its simplicity and parsimoniousness, and the fact that its parameters have not been chosen to replicate the VAR evidence.

E. *Unconditional Performance*

The column labeled "All shocks" in Table 3 shows the unconditional moments produced by the model. The latter replicates well the volatility and cyclicality of the real aggregates. The only exception is hours worked, for which the model generates rather small fluctuations. Notice however that this is a well known feature of standard RBC models with realistic labor supply elasticities. To see this, in the last column of Table 3 we display the unconditional moments produced by the standard RBC model. Indeed, our model actually improves on the RBC model in terms of the volatility of hours. This is because first-moment (TFP and capital quality) shocks produce similar hours fluctuations in both models,[58] and volatility shocks have no effects in the RBC model, as explained in Section IIIB.

[58] Conditional results in the RBC model are available upon request.

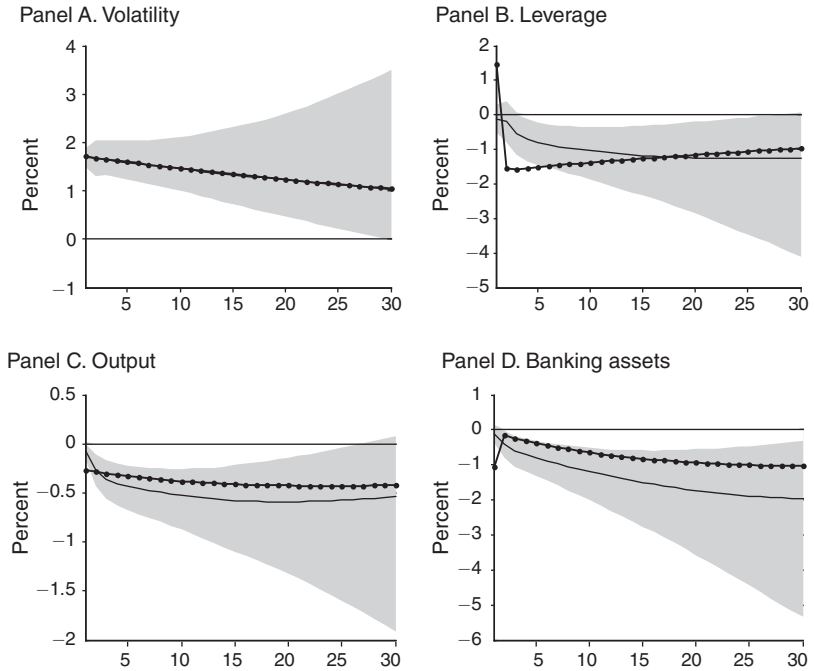

FIGURE 9. IMPULSE RESPONSES OF THE MODEL AND THE DATA

Notes: This figure shows mean impulse response (thin lines) and 95 percent confidence bands for the data, and model impulse response (solid-dotted lines).

Source: US Flow of Funds and Bureau of Economic Analysis. See online data Appendix for details.

The model also replicates well the volatility of observed bank leverage and assets, as well as the procyclicality of leverage with respect to GDP and assets. In both dimensions, volatility shocks play a key role in bringing the model close to the data.

We take a closer look at the model's performance regarding the procyclicality of leverage by computing the cross-correlogram between observed leverage, on the one hand, and GDP and observed assets on the other. Figure 10 displays both conditional and unconditional dynamic cross correlations. The model replicates well the correlations between leverage and GDP or assets at different leads and lags. As the figure makes clear, it is volatility shocks that allow the model to replicate such procyclicality.

Table 3 also reveals that, in the model, marked-to-market book leverage (ϕ_t) is less procyclical than observed book leverage (ϕ_t^{obs}), though it *is* still procyclical, with correlations with GDP of 0.47 and 0.61, respectively. To see why, we display in Figure 11 the responses of both objects to each shock.[59] Following all three contractionary shocks, on impact observed leverage increases *less* than marked-to-market leverage, as the former only partially reflects the impact drop in

[59] For illustration, we also display the response of book leverage when asset price variations are fully ignored, $A_t/(A_t - B_t)$; the latter is sometimes referred to simply as *book leverage*.

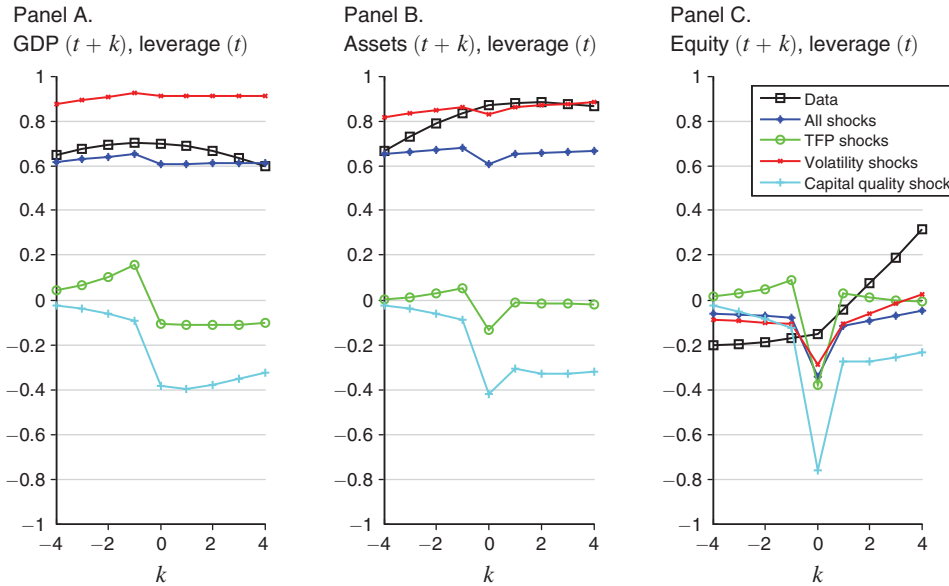

Figure 10. Dynamic Cross-Correlations in the Model and the Data

Source: US Flow of Funds and Bureau of Economic Analysis. See online data Appendix for details.

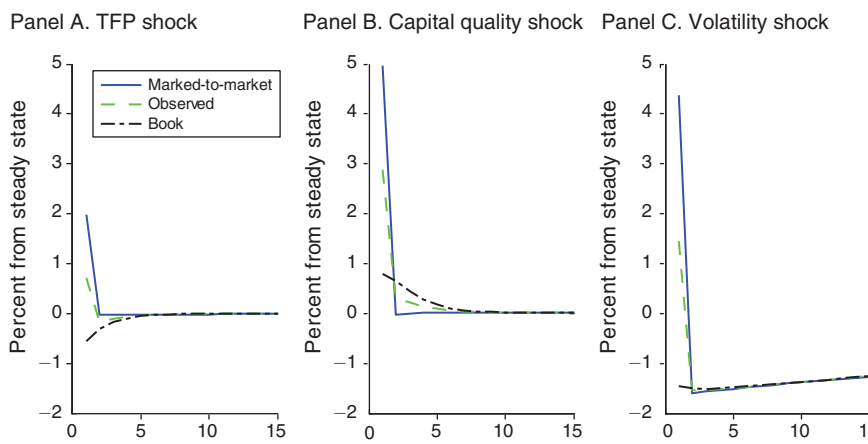

Figure 11. Impulse Responses of Different Measures of Leverage

Note: Deviations with respect to the steady state (percent).

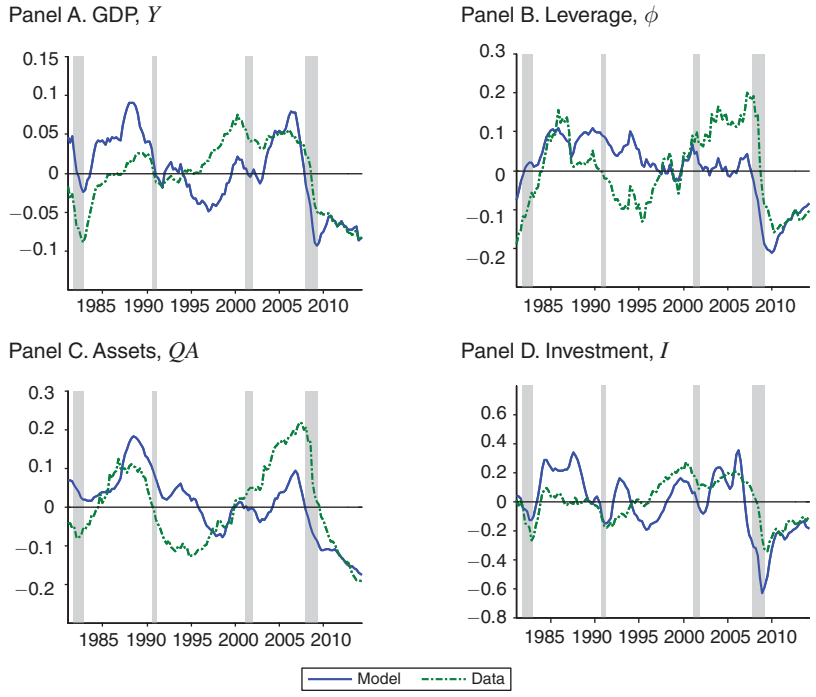

FIGURE 12. HISTORICAL AND MODEL-GENERATED SERIES, COMPLETE MODEL

Notes: All series as log deviations from trend (data) or steady state (model). Shaded areas represent NBER-dated recessions. Model leverage and assets are model counterparts of the observed series $(\phi_t^{obs}, (QA)_t^{obs})$.

Source: US Flow of Funds and Bureau of Economic Analysis. See online Data Appendix for details.

asset prices.[60] It is this countercyclical increase on impact that weakens the procyclicality of marked-to-market leverage vis-à-vis observed leverage. Following the impact period, as asset prices return to normal, both leverage measures behave very similarly. In particular, an increase in volatility produces a large and long-lasting fall in leverage through the volatility-leverage channel.

Figure 12 shows the model-generated series conditional on all three shocks. The model replicates reasonably well the historical evolution of GDP, investment, leverage and assets. Focusing on the 2007–2009 financial crisis period, the model accounts well for the large and protracted fall in GDP relative to trend. The model is also able to replicate the substantial, long-lasting fall in leverage and balance-sheet size suffered by the aggregate intermediary sector during and after the financial crisis. As explained in the previous subsection, the increase in volatility during that period is key in producing such a deleveraging process and protracted fall in GDP.

Finally, we use our model to shed light on how (unobserved) marked-to-market leverage may have evolved during the 2007–2009 financial crisis. The left plot of Figure 13 displays the model-generated series for marked-to-market and observed leverage from 2007 onwards. The right plot shows the model-generated series for

[60] Book leverage actually falls on impact in the case of the TFP or volatility shock, precisely because it fully ignores such asset price drop.

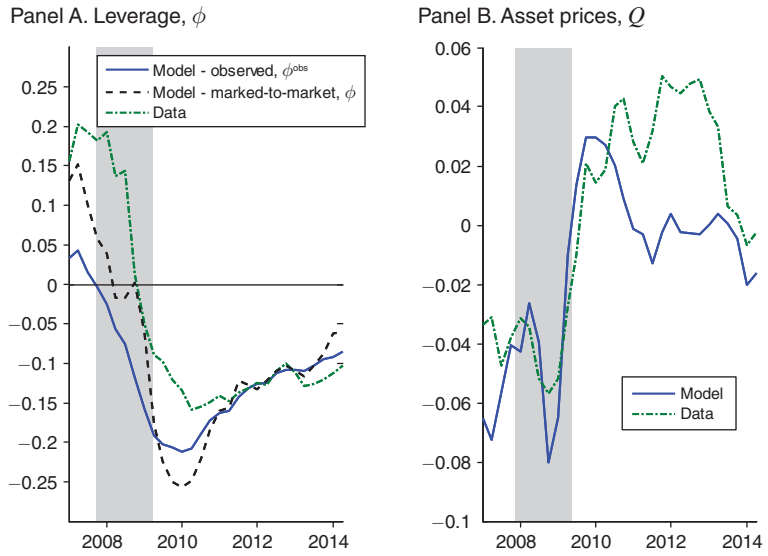

Figure 13. Observed versus Marked-to-Market Leverage during the 2007–2009 Financial Crisis

Note: All series are expressed as log deviations with respect to their trends.

Source: US Flow of Funds and Barclays Capital. See online Data Appendix for details.

bank asset prices, which drive the discrepancy between both measures of leverage. Notice first that the model accounts well for the actual evolution of leverage and asset prices during the crisis. According to the model, the fall in bank asset prices that takes place in the second half of 2008 leads to a countercyclical increase in marked-to-market leverage in that period, through the mechanism explained before. However, following the rapid recovery in asset prices, marked-to-market leverage starts *falling* from 2008:IV onwards, eventually reaching an even lower level than observed leverage. This finding suggests that, when viewed through the lens of our model, marked-to-market and observed leverage behaved similarly during the recent financial crisis.

V. Conclusions and Directions for Future Work

We have presented a general equilibrium model with banks aimed at explaining the main features of the "bank leverage cycle" in the US economy, characterized by large and procyclical fluctuations in the leverage ratio of financial intermediaries. Our main theoretical contribution is to introduce Adrian and Shin's (2014) static, partial equilibrium model of intermediary leverage determination, based on a risk-shifting moral hazard problem, into a dynamic general equilibrium setting. Our results indicate that, unlike TFP or capital quality shocks, volatility shocks generate volatile and procyclical bank leverage, thanks to a volatility-leverage channel in which bank default risk, limited liability, and moral hazard play an important role. The model also replicates well the large and prolonged contractions in leverage, assets, and GDP during and after the 2007–2009 financial crisis.

Consistently with most of the macro-finance literature (Bernanke, Gertler, and Gilchrist 1999; Kiyotaki and Moore 1997; Gertler and Kiyotaki 2010; Christiano, Motto, and Rostagno 2014, etc.), our model only considers short-term debt on the liabilities side of banks' balance sheets. In reality, financial intermediaries are also funded by long-term debt and other more stable sources of funding such as deposits. These debt contracts will be in general more "sticky" across the business cycle, thus reducing the procyclicality of leverage. This may also explain why leverage is more procyclical for intermediaries that rely heavily on short-term funding, such as security broker/dealers. The inclusion of long-term bank debt is therefore a model extension that is worth undertaking in further work.

As in Gertler and Karadi (2011) or Gertler and Kiyotaki (2010), we have also assumed that firms are fully financed with perfectly state-contingent debt (or equity) held by banks. In the absence of a relevant friction in the bank-firm relationship (within the same island), such an assumption is without loss of generality due to the Modigliani-Miller theorem. It would be worthwhile to explore in future work the possibility of introducing frictions that also affect lending relationships between banks and firms. In such a context, if firms issued, say, one-period non-contingent defaultable debt, banks would still be exposed to island-specific risk, only such exposure would be capped at the level of promised corporate debt repayments. Thus, the volatility-leverage channel would continue to operate, although it would probably be quantitatively less powerful.

In our model, banks can recapitalize themselves very quickly following adverse shocks. In this sense, it would be worthwhile to consider possible model modifications that limited the speed of recapitalization. On the one hand, the binding nature of the non-negativity constraint on bank dividends implies that variations in bank asset returns impact directly on net worth. An alternative would be to allow banks to issue equity subject to some convex cost,[61] which would probably deliver a smoother adjustment of equity. On the other hand, our assumption that firms' funding is fully intermediated by banks maximizes the exposure of asset prices, and hence net worth, to banks' balance sheets. In this regard, allowing part of the funding to be provided directly by households (or institutional investors) would tend to limit the response of asset prices and equity, as there would be some redistribution from intermediated to direct financing.[62]

Our analysis attributes a causal role to variations in cross-sectional volatility as a source of fluctuations in intermediary leverage and aggregate economic activity. It is in principle possible that such changes in cross-sectional volatility represent an endogenous response to alternative shocks; see the discussion on this issue in Christiano, Motto, and Rostagno (2014) and the references therein. In this regard, Baker and Bloom (2011) provide some support for the causal nature of volatility shocks.

[61] See, e.g., the model of Jermann and Quadrini (2012), where firms can pay negative dividends (i.e., issue equity) subject to quadratic costs of deviating from some long-run level.

[62] See De Fiore and Uhlig (2015) for a general equilibrium framework that follows a similar approach. See also Adrian, Colla, and Shin (2013).

Finally, this study has adopted a positive focus. We believe that understanding the effects of unconventional monetary policy interventions in this kind of framework may constitute an important topic for future research.

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